CBSE SAMPLE PAPER - 08

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to **[1]**

   a) $(x+1)e^{x+\frac{1}{x}}+c$
   b) $xe^{x+\frac{1}{x}}+c$
   c) $-xe^{x+\frac{1}{x}}+c$
   d) $(x-1)e^{x+\frac{1}{x}}+c$

2. The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12) **[1]**

   a) $cos^{-1}(\frac{2}{3})$
   b) $tan^{-1}(-\frac{2}{3})$
   c) none of these
   d) $cos^{-1}(\frac{3}{2})$

3. If the position vector $\vec{a}$ of the point (5, n) is such that $|\vec{a}|$ = 13, then the value(s) of n can be **[1]**

   a) ± 12
   b) ± 8
   c) Only 12
   d) Only 8

4. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly? **[1]**

   a) $\frac{11}{13}$
   b) $\frac{7}{13}$
   c) $\frac{12}{13}$
   d) $\frac{9}{13}$

5. If for all real triplets (a, b, c), f(x) - a + bx + cx^2; then $\int_0^1$(x) dx is equal to: **[1]**

   a) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$
   b) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$
   c) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$
   d) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$

6. If $P(A)=\frac{1}{4}, P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{5}$, then $P(\bar{B}/\bar{A})$ = ? **[1]**

a) $\frac{37}{45}$

b) $\frac{11}{45}$

c) $\frac{11}{15}$

d) $\frac{37}{60}$

7. The area bounded by the curves y = |x - 1| and y = 1 is given by **[1]**

a) 1

b) $\frac{1}{2}$

c) 2

d) none of these

8. Two-line $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates **[1]**

a) (2, 4, 7)

b) (- 2, 4, 7)

c) (2, - 4, - 7)

d) (2, - 4, 7)

9. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct? **[1]**

a) $\vec{b}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=\vec{0}$

b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$

c) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$

d) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ are mutually perpendicular

10. The degree and order respectively of the differential equation $\frac{dy}{dx}=\frac{1}{x+y+1}$ are **[1]**

a) 1, 2

b) 1, 1

c) 2, 1

d) 2, 2

11. The area of the smaller portion of the circle $x^2 + y^2 = 4$ cut off by the line x = 1 is **[1]**

a) $\frac{4\pi-\sqrt{3}}{3}$

b) $\frac{4\pi-3\sqrt{3}}{3}$

c) none of these

d) $\frac{4\pi+3\sqrt{3}}{3}$

12. $\int_0^1 \frac{x^3}{(1+x^8)}dx=$ **[1]**

a) $\frac{\pi}{4}$

b) $\frac{\pi}{16}$

c) $\frac{\pi}{8}$

d) $\frac{\pi}{2}$

13. Let the f : R → R be defined by f(x) = 2x + cosx, then f : **[1]**

a) is an increasing function

b) is a decreasing function

c) has a minimum at x = π

d) has a maximum, at x = 0

14. The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is **[1]**

a) a diagonal matrix

b) a skew-symmetric matrix

c) a symmetric matrix

d) an upper triangular matrix

15. The equations 2x + 3y = 7, 14x + 21y = 49 has **[1]**

a) infinitely many solutions

b) finitely many solutions

c) a unique solution

d) no solution

[1]

16. If f(x) = $\begin{vmatrix} 2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then, f $(\frac{\pi}{3})$ =

a) 0

b) 1

c) –1

d) 2

17. Which of the following is the principal value branch of $\text{cosec}^{-1}x$? **[1]**

a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

c) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

18. The solution of the differential equation $\left(x^2+1\right)\frac{dy}{dx} + \left(y^2+1\right) = 0$, is **[1]**

a) $y = \frac{1-x}{1+x}$

b) $y = \frac{1+x}{1-x}$

c) $y = 2 + x^2$

d) Y x(x - 1)

19. **Assertion (A):** $y = \frac{e^x + e^{-x}}{2}$ is an increasing function on [0, ∞). **[1]**

Reason (R): $y = \frac{e^x - e^{-x}}{2}$ is an increasing function on (- ∞, ∞).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Determinant of a skew-symmetric matrix of order 3 is zero. **[1]**

Reason (R): For any matrix A, $|A^T| = |A|$ and |-A| = |-A|.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$. **[2]**

22. Solve $(1 + x^2)\sec^2 y dy + 2x \tan y dx = 0$, given that $y = \frac{\pi}{4}$ when x = 1. **[2]**

23. Let D = diag $[d_1, d_2, d_3]$, where none of d_1, d_2, d_3 is 0; prove that D^{-1} = diag $\left[d_1^{-1}, d_2^{-1}, d_3^{-1}\right]$. **[2]**

OR

If A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, verify that A · (Adj A) = (adj A) · A = |A| . I

24. Show that the vectors $2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c}$ and $\vec{a} + \vec{b} - 3\vec{c}$ are non-coplanar vectors. **[2]**

25. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. Out of these bulbs 1%, 2% and 3% of the bulbs produced respectively by A , B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A. **[2]**

Section C

26. Evaluate $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx.$ **[3]**

27. Solve the differential equation: **[3]**

$y + x\frac{dy}{dx} = x - y\frac{dy}{dx}$

OR

In the differential equation show that it is homogeneous and solve it: $x^2 dy + y(x + y)\, dx = 0$.

28. Prove using vectors: The quadrilateral obtained by joining midpoints of adjacent sides of a rectangle is a rhombus. **[3]**

OR

Show that the four points with position vectors $4\hat{i}+8\hat{j}+12\hat{k}, 2\hat{i}+4\hat{j}+6\hat{k}, 3\hat{i}+5\hat{j}+4\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ are coplanar.

29. Evaluate: $\int \frac{8}{(x+2)(x^2+4)}dx$ **[3]**

OR

Evaluate the integral: $\int(4x+1)\sqrt{x^2-x-2}dx$

30. If y = $Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$ **[3]**

31. Find the area of the region bounded by the parabola $y^2 = 4ax$ and the line x = a **[3]**

Section D

32. Minimize Z = x + 2y subject to $2x+y \geqslant 3, x+2y \geqslant 6, x, y \geqslant 0$. Show that the minimum of Z occurs at more than two points. **[5]**

33. Let A = {1, 2, 3} and R = {(a, b):a, b $\in$ A and $|a^2-b^2| \leq 5$. Write R as set of ordered pairs. Mention whether R is **[5]**

i. reflexive
ii. symmetric
iii. transitive

Give reason in each case.

OR

Let n be a fixed positive integer. Define a relation R on Z as follows:

(a, b) $\in$ R $\Leftrightarrow$ a - b is divisible by n. Show that R is an equivalence relation on Z.

34. Find the length shortest distance between the lines: $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ and $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ **[5]**

OR

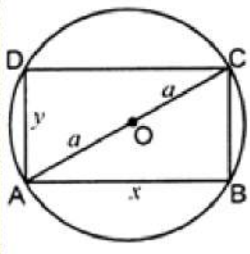
Find the coordinates of the point where the line through the points A (3, 4, 1) and B(5, 1, 6) crosses the XY-plane.

35. Differentiate $\sin^{-1}(2x\sqrt{1-x^2})$ with respect to $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$, if $x \in (1/\sqrt{2}, 1)$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Find the perimeter of rectangle in terms of any one side and radius of circle.

(ii) Find critical points to maximize the perimeter of rectangle?

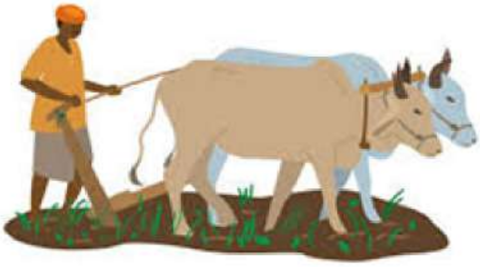
(iii) Check for maximum or minimum value of perimeter at critical point.

OR

If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

37. **Read the text carefully and answer the questions:** **[4]**

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

October sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

(i) Find the combined sales of Masoor in September and October, for farmer Girish.

(ii) Find the combined sales of Urad in September and October, for farmer Ankit.

(iii) Find a decrease in sales from September to October.

OR

If both the farmers receive 2% profit on gross sales, then compute the profit for each farmer and for each variety sold in October.

38. **Read the text carefully and answer the questions:** **[4]**

In a bilateral cricket series between India and South Africa, the probability that India wins the first match is 0.6. If India wins any match, then the probability that it wins the next match is 0.4, otherwise, the probability is 0.3. Also, it is given that there is no tie in any match.

(i) Find the probability that India won the second match, if India has already loose the first match.

(ii) Find the probability that India losing the third match, if India has already lost the first two matches.

Solution

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Class 12 - Mathematics

Section A

1. **(b)** $xe^{x+\frac{1}{x}}+c$

Explanation: $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$

$=\int e^{x+\frac{1}{x}}dx+\int x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$

$=\int e^{x+\frac{1}{x}}dx+xe^{x+\frac{1}{x}}-\int\frac{d}{dx}(x)e^{x+\frac{1}{x}}dx$

$=\int e^{x+\frac{1}{x}}dx+xe^{x+\frac{1}{x}}-\int e^{x+\frac{1}{x}}dx\ \left(\because\int\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx=e^{x+\frac{1}{x}}\right]$

$=\int e^{x+\frac{1}{x}}dx+xe^{x+\frac{1}{x}}-\int ex^{x+\frac{1}{x}}dx=xe^{x+\frac{1}{x}}+c$

2. **(a)** $cos^{-1}(\frac{2}{3})$

Explanation: The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12)

Direction ratios of the line joining the points A(3, 1, 4), B(7, 2, 12) is <x2-x1 , y2-y1, z2-z1> = < 7-3 , 2-1 , 12-4> = <4 ,1 ,8>

Now as the angle between two lines having direction ratios <a1,b1,c1> and <a2,b2,c2> is given by

$\text{Cos}^{-1}\frac{a1a2+b1b2+c1c2}{\sqrt{a1^2+b1^2+c1^2}\sqrt{a2^2+b2^2+c2^2}}$

Using the vuales we have

$\cos^{-1}\frac{2\times4+2\times1+1\times8}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}=\cos^{-1}\frac{18}{27}=\cos^{-1}\frac{2}{3}$

3. **(a)** ±12

Explanation: We have,

$\vec{a}=5\hat{i}+n\hat{j}$

$\therefore|\mathbf{a}|=\sqrt{25+n^2}=13$

$\Rightarrow 25+n^2=169$

$\Rightarrow n^2=169-25=144$

$\Rightarrow n=\pm12$

4. **(c)** $\frac{12}{13}$

Explanation: Let E_1 and E_2 are events that the student knows the answer and the student guesses respectively.

$P(E_1)=\frac{3}{4},P(E_2)=\frac{1}{4}$.

Let A = event that the student answers correctly.

$P(E_1)=\frac{3}{4},P(E_2)=\frac{1}{4}$

$P(A/E_1)=1$ (as it is sure event)

$P(A/E_2)=\frac{1}{4}$

$P(E_1/A)=\frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}=\frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}}=\frac{12}{13}$

5. **(b)** $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

Explanation: $\int_0^1(a+bx+cx^2)dx=ax+\frac{bx^2}{2}+\frac{cx^3}{3}\Big|_0^1=a+\frac{b}{2}+\frac{c}{3}$

Now, f(1) = a + b + c, f(0) = a and $f\left(\frac{1}{2}\right)=a+\frac{b}{2}+\frac{c}{4}$

Now, $\frac{1}{6}\left(f(1)+f(0)+4f\left(\frac{1}{2}\right)\right)$

$=\frac{1}{6}\left(a+b+c+a+4\left(a+\frac{b}{2}+\frac{c}{4}\right)\right)$

$=\frac{1}{6}(6a+3b+2c)=a+\frac{b}{2}+\frac{c}{3}$

Hence, $\int_0^1 f(x)=\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

6. **(a)** $\frac{37}{45}$

Explanation: $P(A)=\frac{1}{4},P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{5}$

$P(\overline{(B}/\bar{A}) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{A})}$

$= \frac{1-P(A\cup B)}{1-P(A)} = \frac{1-\left(\frac{1}{4}+\frac{1}{3}-\frac{1}{5}\right)}{1-\frac{1}{4}}$

$\Rightarrow P(\bar{B}/\bar{A}) = \frac{37}{45}$.

Which is the required solution.

7. **(a)** 1

Explanation: The given curves are : (i) y = x – 1 , x > 1 . (ii) y = - (x – 1) , x < 1. (iii) y = 1 these three lines enclose a triangle whose area is : $\frac{1}{2}$.base.height = $\frac{1}{2}$.2 .1 = 1 sq. unit.

8. **(c)** (2, - 4, - 7)

Explanation: Let $\frac{x-3}{1}$ = $\frac{y+1}{3}$ = $\frac{z-6}{-1}$ = a (say). Then, any point on this line is of form P(a + 3, 3a -1, -a + 6)

Similarly, any point on the line. $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ = b (say), is of the form Q(7b - 5, -6b + 2, 4b + 3)

Now, if the lines are intersect, then P = Q for some a and b.

$\Rightarrow$ a + 3 = 7b - 5

3a - 1 = -6b + 2 and -a + 6 = 4b + 3

$\Rightarrow$ a - 7b = -8, a + 2b = 1 and a + 4b = 3

On solving a - 7b = -8 and a + 2b = 1, we get b = 1 and a = -1, which also satisfy a + 4b = 3

$\therefore$ P = Q $\equiv$ (2, -4, 7) for a = -1 and b = 1

Thus, coordinates of point R are (2, -4, 7) and reflection of R in xy-plane is (2, -4, -7)

9. **(b)** $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$

Explanation: Since, $\vec{a}, \vec{b}, \vec{c}$ are unit vectors and $\vec{a}+\vec{b}+\vec{c}$ = 0, then $\vec{a}, \vec{b}, \vec{c}$ represent an equilateral triangle.

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$.

10. **(b)** 1, 1

Explanation: 1, 1

11. **(b)** $\frac{4\pi-3\sqrt{3}}{3}$

Explanation: Required area:

$= 2\int_1^2 \sqrt{4-x^2}dx$

$= 2\left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$

$= 2\left[\frac{4}{2}\sin^{-1}(1) - \frac{\sqrt{3}}{2} - \frac{4}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$

$= \frac{4\pi-3\sqrt{3}}{3}$

12. **(b)** $\frac{\pi}{16}$

Explanation: Let, $x^4 = t$

Differentiating both side with respect to t

$4x^3\frac{dx}{dt} = 1$

$\Rightarrow x^3dx = \frac{1}{4}dt$

At x = 0, t = 0

At x = 1, t = 1

$y = \frac{1}{4}\int_0^1 \frac{1}{1+t^2}dt$

$= \frac{1}{4}\left(\tan^{-1}t\right)_0^1$

$= \frac{1}{4}\left(\tan^{-1}1 - \tan^{-1}0\right)$

$= \frac{\pi}{16}$

13. **(a)** is an increasing function

Explanation: We have, f(x) = 2x + cos x

$\therefore f'(x) = 2 - \sin x > 0, \forall x$

Hence, f(x) is an increasing function

14. **(b)** a skew-symmetric matrix

Explanation: A = $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$

A^T = $\begin{bmatrix} 0 & -5 & 7 \\ 5 & 0 & -11 \\ -7 & 11 & 0 \end{bmatrix}$

-A = $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$

$\because A^T = -A$

Then, the given matrix is a skew–symmetric matrix.

15. **(a)** infinitely many solutions

Explanation: For infinitely many solutions ,$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. , for given system of equations we have : $\frac{2}{14} = \frac{3}{21} = \frac{7}{49}$.

16. **(c)** –1

Explanation: $\begin{vmatrix} 2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$

Put x = $\frac{\pi}{3}$, $\begin{vmatrix} 2\cos\frac{\pi}{3} & 1 & 0 \\ 1 & 2\cos\frac{\pi}{3} & 1 \\ 0 & 1 & 2\cos\frac{\pi}{3} \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2.\frac{1}{2} & 1 & 0 \\ 1 & 2.\frac{1}{2} & 1 \\ 0 & 1 & 2.\frac{1}{2} \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \Rightarrow 1(0) - 1(1) = -1$

17. **(b)** $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Explanation: We know that the principal value branch of cosec^{-1} x is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

18. **(a)** $y = \frac{1-x}{1+x}$

Explanation: $y = \frac{1-x}{1+x}$

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion Let f(x) = $\frac{e^x+e^{-x}}{2}$

$\Rightarrow$ f'(x) = $\frac{e^x-e^{-x}}{2} = \frac{1}{2}\left(e^x - \frac{1}{e^x}\right)$

= $\frac{1}{2}\left(\frac{e^{2x}-1}{e^x}\right)$ ...(i)

Now, for x $\geq$ 0, we have

2x $\geq 0 \Rightarrow e^{2x} \geq e^0$ [$\because e^x$ is an increasing function]

$\Rightarrow e^{2x} \geq 1$

Also, for x $\geq$ 0

$\Rightarrow e^x \geq 1$

$\therefore$ From Eq. (i), we have

f'(x) = $\frac{1}{2}\left(\frac{e^{2x}-1}{e^x}\right) \geq 0$

So, f(x) is an increasing function on [0, ∞).

Reason: Let g(x) = $\frac{e^x-e^{-x}}{2}$

$\Rightarrow$ g'(x) = $\frac{e^x+e^{-x}}{2} > 0$ [$\therefore e^x$ and e^{-x} both are greater than zero in (-∞, ∞)]

So, g(x) is an increasing function on (-∞, ∞).

Hence, both Assertion and Reason are true.

20. **(c)** A is true but R is false.

Explanation: Assertion: Determinant of a skew-symmetric matrix of odd order is zero.

$\therefore$ Assertion is true.

Reason: For any matrix A, $|A^T| = A$

and $|-A| = |A|$ [when A is of even order]

and $|-A| = -|A|$ [when A is of odd order]

$\therefore$ Reason is false.

Section B

21. Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

22. Here, $(1 + x^2) \sec^2 y\, dy + 2x \tan y\, dx = 0$,

Given that, $y = \frac{\pi}{4}$ when x = 1

$\Rightarrow (1 + x^2)\sec^2 y\, dy + 2x\tan y\, dx = 0$

$\Rightarrow \frac{\sec^2 y}{\tan y} dy + \frac{2x}{1+x^2} dx = 0$

$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy + \int \frac{2x}{1+x^2} dx = 0$

$\Rightarrow \log \tan y + \log (1 + x^2) = \log c$

for $y = \frac{\pi}{4}$, x, = 1

We have, $0 + \log 2 = \log c$,

c = 2, Therefore,the required particular solution is:-

$\therefore \tan y \left(1 + x^2\right) = 2$

23. We are Given that ; $D = \text{diag}\ [d_1, d_2, d_3]$

It is also given that $d_1 \neq 0, d_2 \neq 0, d_3 \neq 0$

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

A diagonal matrix $D = \text{diag}(d_1, d_2, \ldots d_n)$ is invertible iff all diagonal entries are non zero, i.e. $d_i \neq 0$ for $1 \leq i \leq n$

If D is invertible then $D^{-1} = \text{diag}\left(d_1^{-1}, \ldots d_n^{-1}\right)$

By the Inverting Diagonal Matrix Theorem, which states that

Here, it is given that $d_1 \neq 0, d_2 \neq 0, d_3 \neq 0$

$\therefore$ D is invertible

$\Rightarrow D^{-1} = \text{diag}\left[d_1^{-1}, d_2^{-1}, d_3^{-1}\right]$

Hence Proved

OR

Given: $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = (15 - 7) = 8 \neq 0$

The cofactors of the elements of IAI are given by

$A_{11} = 5, A_{12} = -7$;

$A_{21} = -1, A_{22} = 3$

$\therefore$ (adj A) = $\begin{bmatrix} 5 & -7 \\ -1 & 3 \end{bmatrix}' = \begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

$\therefore A \cdot (\text{Adj } A) = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

$= \begin{bmatrix} 15 - 7 & -3 + 3 \\ 35 - 35 & -7 + 15 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$= 8 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 8I = |A| \cdot I\ [\because |A| = 8]$

And, (adj A) · A = $\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$

$= \begin{bmatrix} 15-7 & 5-5 \\ -21+21 & -7+15 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$= 8 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = 8I |A| · I [∵ |A| = 8]

Therefore, A · (Adj A) = (adj A) · A = |A| · I

Hence proved.

24. Suppose, if possible, the given vectors be coplanar. Then one of the given vectors is expressible in terms of the other two.

Let $2\vec{a} - \vec{b} + 3\vec{c} = x(\vec{a} + \vec{b} - 2\vec{c}) + y(\vec{a} + \vec{b} - 3\vec{c})$ form some scalars x and y

$\Rightarrow 2\vec{a} - \vec{b} + 3\vec{c} = (x+y)\vec{a} + (x+y)\vec{b} + (-2x-3y)\vec{c}$

⇒ 2 = x + y, -1 = x + y and 3 = -2x - 3y

Solving, first and third of these equations, we obtain x = 9 and y = -7.

Clearly, these values do not satisfy the third equation.

Therefore, the given vectors are not coplanar.

25. Let A: bulb manufactured from machine A

B :bulb Manufactured from machine B

C :bulb Manufactured from machine C

D : Defective bulb

We want to find $P(\frac{B}{AD})$ i.e. probability of selected defective bulb is from machine A.

Therefore,by Baye's theorm,we have,

$P(\frac{B}{AD}) = \frac{\mathrm{P(A) \cdot P(D|A)}}{\mathrm{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)}}$

$= \frac{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right)}{\left(\frac{60}{100}\right)\left(\frac{1}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{2}{100}\right) + \left(\frac{10}{100}\right)\left(\frac{3}{100}\right)}$

$= \frac{6}{15} = \frac{2}{5}$

Conclusion: Therefore, the probability of selected defective bulb is from machine A is $\frac{2}{5}$

Section C

26. Let $I = \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$

$\Rightarrow I = \int \frac{(\sin^2 x)^3 + (\cos^2 x)^3}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\left[\begin{array}{l} (\sin^2 x + \cos^2 x)^3 \\ -3\sin^2 x \cos^2 x (\sin^2 x + \cos^2 x) \end{array}\right]}{\sin^2 x \cos^2 x} dx$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$= \int \frac{(1)^3 - 3\sin^2 x \cos^2 x}{\sin^2 x \cos^2 x} dx$

[∵$\sin^2$x + $\cos^2$ x = 1]

$= \int \frac{1}{\sin^2 x \cos^2 x} dx - 3 \int \frac{\sin^2 x \cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx - 3 \int 1 dx$

$= \int \left[\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x}\right] dx - 3 \int 1 dx$

$= \int \left(\sec^2 x + \cot^2 x\right) dx - 3 \int 1 dx$

$= \int \sec^2 x dx + \int \cot^2 x dx - 3 \int 1 dx$

$= tan\, x - cotx - 3x + C$

27. The given differential equation is,

y+ $x\frac{dy}{dx}$ = x - $y\frac{dy}{dx}$

(x + y) $\frac{dy}{dx}$ = x - y

$\frac{dy}{dx} = \frac{x-y}{x+y}$

Clearly, it is homogeneous equation of degree 1 put y = vx , therefore, we have,

v + $x\frac{dv}{dx} = \frac{x-vx}{x+vx}$

or v + $x\frac{dv}{dx} = \frac{1-v}{1+v}$

or $x\frac{dv}{dx} = \frac{1-v}{1+v} - v$

$= \frac{1-v-v-v^2}{1+v}$

or $x\frac{dv}{dx} = \frac{1-2v-v^2}{1+v}$

$\therefore \frac{1+v}{1-2v-v^2}dv = \frac{dx}{x}$

Integrating both sides,we have,

$\int \frac{1+v}{1-2v-v^2}dv = \int \frac{dx}{x}$

Let (1 + v) = A$\frac{d}{dx}$(1 - 2v - v^2) + B

$\therefore$ 1 + v = A(-2 - 2v) + B

When v = -1,

1 - 1 = B or B = 0

When v = 0,

1 = -2A

or A = -$\frac{1}{2}$

$\therefore \int \frac{-\frac{1}{2}(-2-2v)}{1-2v-v^2}dv = \int \frac{dx}{x}$

or -$\frac{1}{2}$log |1 - 2v - v^2| = log |x| + log C

$\Rightarrow$ log C = log|x| + $\frac{1}{2}$log |1 - 2v - v^2|

log C' = log $\left(x\sqrt{1-2v-v^2}\right)$

C' = $\frac{1}{C}$

C' = $x\sqrt{1-2v-v^2}$

Substitute v = $\frac{y}{x}$

C' = $x\sqrt{1-2\left(\frac{y}{x}\right)-\frac{y^2}{x^2}}$

= $\frac{x}{x}\sqrt{x^2-2yx-y^2}$

= $\sqrt{x^2-2yx-y^2}$

or $C'^2 = x^2 - 2yx - y^2$

$\therefore$ Solution is $C_1 = x^2 - 2yx - y^2 (C_1 = C'^2)$

This is the required solution.

OR

The given differential equation is,

$x^2dy + y(x + y)\, dx = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y(x+y)}{x^2} = -\left(\frac{y}{x}+\frac{y^2}{x^2}\right)$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \mathrm{v} + \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} = -\left(\frac{\mathrm{vx}}{\mathrm{x}}+\frac{(\mathrm{vx})^2}{\mathrm{x}^2}\right)$

$\Rightarrow \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}$ = -v-v^2-v = -2v -v^2

$\Rightarrow \frac{\mathrm{dv}}{2\mathrm{v}+\mathrm{v}^2} = -\frac{\mathrm{dx}}{\mathrm{x}}$

Integrating both the sides we get:

$\int \frac{\mathrm{dv}}{2\mathrm{v}+\mathrm{v}^2} = -\int \frac{\mathrm{dx}}{\mathrm{x}} + \log|\mathrm{c}|$

$\Rightarrow \int \frac{dv}{1+2v+v^2-1}$ = -ln |x| + ln|c|

$\Rightarrow \int \frac{dv}{(v+1)^2-1^2}$ + ln|x| = ln|c|

$\Rightarrow \frac{1}{2}\ln\left|\frac{v+1-1}{v+1+1}\right|$ + ln|x| = ln|c|

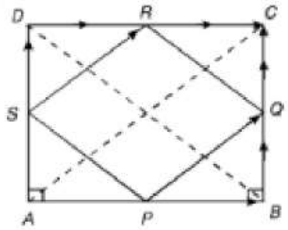
$\Rightarrow \ln\left|\frac{\mathrm{v}+1-1}{\mathrm{v}+1+1}\right|$ + 2 ln|x| = 2 ln|c|

Re-substituting the value of y = vx we get

$\Rightarrow \ln\left|\frac{\frac{y}{x}}{\frac{y}{x}+2}\right|$ + lnx^2 = ln$|c|^2$

$\Rightarrow \ln\left|\frac{y}{y+2x}\right| + \text{In}x^2 = \text{In}|c|^2$

$\Rightarrow x^2y = c^2(y+2x)$, which is the required solution.

28. 

ABCD be rectangle.

Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively,

Now

$\overline{PQ} - \overline{P\bar{B}} + \overline{BQ} - \frac{1}{2}(\overline{AB} + \overline{B\bar{C}}) - \frac{1}{2}\overline{AC}$ ....(i)

$\overline{\text{SR}} - \overline{\text{SD}} + \overrightarrow{\text{DR}} - \frac{1}{2}(\overrightarrow{\text{AD}} + \overline{\text{DC}}) - \frac{1}{2}\overline{\text{AC}}$ ....(ii)

From (i) and (ii) we have

$\overrightarrow{PQ} = \overrightarrow{SR}$ i, e. sides PQ and SR are equal and parallel

PQRS is a parallelogram.

$(PQ)^2 = \overline{PQ}\cdot\overline{PQ} = (P\bar{B} + \overline{BQ})\cdot(\overline{PB} + \overline{BQ}) - PE^2 + BQ^2$ .... (iii)

$(PS)^2 - \overline{PS}\cdot\overline{PS} - (\overline{PA} + \overline{PS})\cdot(\overline{PA} + \overline{PS}) - PA|^2 + AS|^2 - PEP^2 + |BQ|^2$ ....(iv)

From (iii) and (iv) we get,

$(PQ)^2 = (PQ)^2$ i. e. $PQ = PS$

= The adjacent sides of PQRS are equal.

$PQRS$ is a rhombus.

OR

Let $\overrightarrow{a} = 4\hat{i} + 8\hat{j} + 12\hat{k}$

$\overrightarrow{b} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$\overrightarrow{c} = 3\hat{i} + 5\hat{j} + 4\hat{k}$

$\overrightarrow{d} = 5\hat{i} + 8\hat{j} + 5\hat{k}$

Now, $[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 4 & 8 & 12 \\ 2 & 4 & 6 \\ 3 & 5 & 4 \end{vmatrix}$

= 4(16 - 30) -8(8 - 18) + 12(10 - 12)

= 4(-14) - 8(-10) + 12(-2)

= -56 + 80 - 24

= -80 + 80

= 0

So, $\hat{a}, \hat{b},$ & $\hat{c}$ are Co-planar

Also, $\left[b \quad \overrightarrow{c} \quad \overrightarrow{d}\right] = \begin{vmatrix} 2 & 4 & 6 \\ 3 & 5 & 4 \\ 5 & 8 & 5 \end{vmatrix}$

= 2(25 - 32) -4(15 - 20) + 6(24 - 25)

= 2(-7) - 4(-5) - 6

= -14 - 6 + 20

= 0

i.e., $\overrightarrow{b}$, $\overrightarrow{c}$ & $\overrightarrow{d}$ are co-planar

Hence, $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ & $\overrightarrow{d}$ are co-planar

29. Solving this by using partial fractions

Let $\frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$ ......(i)

Then, 8 = A $(x^2 + 4)$ + (Bx + C) (x + 2) ......(ii)

Putting x + 2 = 0 i.e. x = - 2 in (ii), we get

$8 = 8A \Rightarrow A = 1$

Putting x = 0 and 1 respectively in (ii), we get

8 = 4A + 2C and 8 = 5A + 3B + 3C

Solving these equations, we obtain

A = 1, C = 2 and B = - 1

Substituting the values of A, B and C in (i), we obtain

$\frac{8}{(x+2)(x^2+4)} = \frac{1}{x+2} + \frac{-x+2}{x^2+4}$

$\therefore I = \int \frac{8}{(x+2)(x^2+4)} dx$

$\Rightarrow I = \int \frac{1}{x+2} dx + \int \frac{-x+2}{x^2+4} dx$

$\Rightarrow I = \int \frac{1}{x+2} dx - \int \frac{x}{x^2+4} dx + 2\int \frac{1}{x^2+4} dx$

$\Rightarrow I = \log |x + 2| - \frac{1}{2} \int \frac{1}{t} dt + 2 \times \frac{1}{2} \tan^{-1} \frac{x}{2} + C$

$= \log |x + 2| - \frac{1}{2} \log (x^2 + 4) + \tan^{-1} \frac{x}{2} + C$

OR

Let the given integral be,

$I = \int (4x+1)\sqrt{x^2-x-2}dx$

Also, $4x + 1 = \lambda \frac{d}{dx}\left(x^2-x-2\right) + \mu$

$\Rightarrow 4x + 1 = \lambda(2x - 1) + \mu$

$\Rightarrow 4x + 1 = (2\lambda)x + \mu - \lambda$

Equating coefficient of like terms

$2\lambda = 4$

$\Rightarrow \lambda = 2$

And

$\mu - \lambda = 1$

$\Rightarrow \mu - 2 = 1$

$\Rightarrow \mu = 3$

$\therefore I = \int [2(2x-1)+3]\sqrt{x^2-x-2}dx$

$= 2\int (2x-1)\sqrt{x^2-x-2}dx + 3\int \sqrt{x^2-x-2}dx$

$= 2\int (2x-1)\sqrt{x^2-x-2}dx + 3\int \sqrt{x^2-x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2}dx$

$= 2\int (2x+1)\sqrt{x^2-x-2}dx + 3\int \sqrt{\left(x-\frac{1}{2}\right)^2-2-\frac{1}{4}}dx$

$= \int (2x-1)\sqrt{x^2-x-2}dx + 3\int \sqrt{\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2}dx$

Let $x^2 - x - 2 = t$

$\Rightarrow (2x - 1)dx = dt$

$\therefore I = 2\int \sqrt{t}dt + 3\left[\left(\frac{x-\frac{1}{2}}{2}\right)\sqrt{\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2} - \frac{\left(\frac{3}{2}\right)^2}{2}\log\left|\left(x-\frac{1}{2}\right)+\sqrt{x^2-x-2}\right|\right]$

$= 2\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right] + \frac{3}{4}(2x-1)\sqrt{x^2-x-2} - \frac{27}{8}\log\left|\left(x-\frac{1}{2}\right)+\sqrt{x^2-x-2}\right| + C$

$= \frac{4}{3}\left(x^2-x-2\right)^{\frac{3}{2}} + \frac{3}{4}(2x-1)\sqrt{x^2-x-2} - \frac{27}{8}\log\left|\left(x-\frac{1}{2}\right)+\sqrt{x^2-x-2}\right| + C$

30. Given: $y = Ae^{mx} + Be^{nx}$ ....(i)

To prove: $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

$\therefore \frac{dy}{dx} = Ae^{mx}\frac{d}{dx}(mx) + Be^{nx}\frac{d}{dx}(nx)$ $\left[\because \frac{d}{dx}e^{f(x)} = e^{f(x)}\frac{d}{dx}f(x)\right]$

$\Rightarrow \frac{dy}{dx} = Ame^{mx} + Bne^{nx}$ ....(ii)

Again $\frac{d^2y}{dx^2} = Ame^{mx}.m + Bne^{nx}.n$

$= Am^2e^{mx} + Bn^2e^{nx}$ ...(iii)

Now, L.H.S.= $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$

$= Am^2e^{mx} + Bn^2e^{nx} - (m + n)(Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$

$= Am^2e^{mx} + Bn^2e^{nx} - Am^2e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2e^{nx} + Amne^{mx} + Bmne^{nx}$

= 0

= R.H.S. Hence proved.

31. We have to find the area of the region bounded by

x = a ...(i)

and $y^2 = 4$ ax ...(ii)

Equation (i) represents a line parallel to y-axis and equation (ii) represents a parabola. we slice this information into rectangular strip of width =dx, Length = y - 0 = y

Area of rectangle $= y \triangle x$

This approximating rectangle can move from x = 0 to x = a.

Required area = Twice area of Region OCBO

= 2(Region OABO)

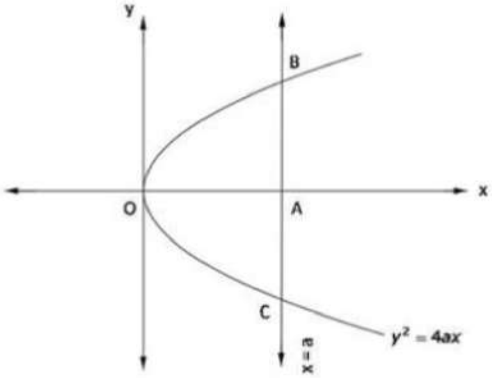
$= 2 \int_0^a \sqrt{4ax} dx$

$= 2.2\sqrt{a} \int_0^a \sqrt{x} dx$

$= 4\sqrt{a} \cdot \left(\frac{2}{3} x\sqrt{x}\right)_0^a$

$= 4\sqrt{a} \cdot \left(\frac{2}{3} a\sqrt{a}\right)$

Required area $= \frac{8}{3} a^2$ square units.



Section D

32. Consider $2x + y \geqslant 3$

Let 2x + y = 3 ⇒ y = 3 - 2x

x	0	1	–1
y	3	1	5

(0, 0) is not contained in the required half plane as (0, 0) does not satisfy the inequation $2x + y \geqslant 3$.

Again $x + 2y \geqslant 6$

Let x + 2y = 6

$\Rightarrow \frac{x}{6} + \frac{y}{3} = 1$

Here also (0, 0) does not contain the required half plane. The double shaded region XABY is the solution set. Its corners are A (6, 0) and B (0, 3).

At A (6, 0) Z = 6 + 0 = 6

At B (0, 3) Z = 0 + 2 × 3 = 6

Therefore, at both points the value of Z = 6 which is minimum. In fact at every point on the line AB makes Z = 6 which is also minimum.

33. Given that

Let A = {1, 2, 3} and R = {(a, b) : a, b ∈ A and $|a^2 - b^2| \leq 5$

Put a = 1, b = 1 $|1^2 - 1^2| \leq 5, (1, 1)$ is an ordered pair.

Put a = 1, b = 2 $|1^2 - 2^2| \leq 5, (1, 2)$ is an ordered pair.

Put a = 1, b = 3 $|1^2 - 3^2| > 5, (1, 3)$ is not an ordered pair.

Put a = 2, b = 1 $|2^2 - 1^2| \leq 5, (2, 1)$ is an ordered pair.

Put a = 2, b = 2 $|2^2 - 2^2| \leq 5, (2, 2)$ is an ordered pair.

Put a = 2, b = 3 $|2^2 - 3^2| \leq 5, (2, 3)$ is an ordered pair.

Put a = 3, b = 1 $|3^2 - 1^2| > 5, (3, 1)$ is not an ordered pair.

Put a = 3, b = 2 $|3^2 - 2^2| \leq 5, (3, 2)$ is an ordered pair.

Put a = 3, b = 3 $\left|3^2 - 3^2\right| \leq 5, (3,3)$ is an ordered pair.

R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

i. For (a, a) $\in$ R

   $\left|a^2 - a^2\right| = 0 \leq 5$. Thus, it is reflexive.

ii. Let (a, b) $\in$ R

   $(a, b) \in R, \left|a^2 - b^2\right| \leq 5$

   $\left|b^2 - a^2\right| \leq 5$

   (b, a) $\in$ R

   Hence, it is symmetric

iii. Put a = 1, b = 2, c = 3

   $\left|1^2 - 2^2\right| \leq 5$

   $\left|2^2 - 3^2\right| \leq 5$

   But $\left|1^2 - 3^2\right| > 5$

   Thus, it is not transitive

OR

R = {(a, b): a - b is divisible by n} on Z.

Now,

Reflexivity: Let a $\in$ Z

$\Rightarrow$ a - a = 0 $\times$ n

$\Rightarrow$ a - a is divisible by n

$\Rightarrow$ (a, a) $\in$ R

$\Rightarrow$ R is reflexive

Symmetric: Let (a, b) $\in$ R

$\Rightarrow$ a - b = np for some p $\in$ Z

$\Rightarrow$ b - a = n(-p)

$\Rightarrow$ b - a is divisible by n

$\Rightarrow$ (b, a) $\in$ R

$\Rightarrow$ R is symmetric

Transitive: Let (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow$ a - b = np and b - c = nq for some p, q $\in$ Z

$\Rightarrow$ a - c = n(p + q)

$\Rightarrow$ a - c is divisible by n

$\Rightarrow$ (a, c) $\in$ R

$\Rightarrow$ R is transitive

Thus, R being reflexive, symmetric and transitive on Z.

Hence, R is an equivalence relation on Z

34. Here,it is given equations of lines

$L_1 : \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$

$L_2 : \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$

Direction ratios of L_1 and L_2 are (2, 1, -3) are (2, -7, 5) respectively.

Suppose, general point on line L_1 is P = (x_1, y_1, z_1)

$x_1 = 2s - 1$, $y_1 = s + 1$, $z_1 = -3s + 9$

and let general point on line L_2 is Q = (x_2, y_2, z_2)

$x_2 = 2t + 3$, $y_2 = -7t - 15$, $z_2 = 5t + 9$ then ,we get

$\therefore \overrightarrow{PQ} = (x_2 - x_1)\,\hat{\imath} + (y_2 - y_1)\,\hat{\jmath} + (z_2 - z_1)\,\hat{k}$

$= (5t + 9 - 2s + 1)\hat{i} + (-7t - 15 - s - 1)\hat{\jmath} + (5t + 9 + 3s - 9)\hat{k}$

$\therefore \overrightarrow{PQ} = (5t - 2s + 10)\hat{\mathrm{i}} + (-7t - s - 16)\hat{\jmath} + (5t + 3s)\hat{k}$

Direction ratios of $\overrightarrow{PQ}$ are ((5t - 2s + 10), (-7t - s - 16), (5t + 3s))

PQ will be the shortest distance if it is perpendicular to both the given lines

Thus, by the condition of perpendicularity, 2(5t - 2s + 10) + 1(-7t - s - 16) - 3(5t - 3s) = 0 and

2(5t - 2s + 10) -7(-7t - s - 16) + 5(5t + 3s) = 0

$\Rightarrow$ 10t - 4s + 20 - 7t - s - 16 - 15t - 9s = 0 and

10t - 4s + 20 + 49t + 7s + 112 + 25t + 15s = 0

$\Rightarrow$ -12t - 14s = -4 and 84t + 18s = -132

Solving above two equations, we obtain

t = -2 and s = 2

thus,

P = (3, 3, 3) and Q = (-1, -1, -1)

Now, distance between points P and Q is

$d=\sqrt{(3+1)^2+(3+1)^2+(3+1)^2}$

$=\sqrt{(4)^2+(4)^2+(4)^2}$

$=\sqrt{16+16+16}$

$=\sqrt{48}$

$=4\sqrt{3}$

Thus, the shortest distance between two given lines is

$d=4\sqrt{3}$ units

Now, equation of line passing through points P and Q is

$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$

$\therefore \frac{x-3}{3+1}=\frac{y-3}{3+1}=\frac{z-3}{3+1}$

$\therefore \frac{x-3}{4}=\frac{y-3}{4}=\frac{z-3}{4}$

$\therefore$ x - 3 = y - 3 = z - 3

$\Rightarrow$ x = y = z

Therefore, equation of line of shortest distance between two given lines is x = y = z

OR

The vector equation of the line through point A and B is

$\vec{r}=3\hat{i}+4\hat{j}+\hat{k}+\lambda\left[(5-3)\,\hat{i}+(1-4)\,\hat{j}+(6-1)\,\hat{k}\right]$

$\Rightarrow \vec{r}=3\hat{i}+4\hat{j}+\hat{k}+\lambda\left(2\hat{i}-3\hat{j}+5\hat{k}\right)$

$\vec{r}=(3+2\lambda)\hat{i}+(4-3\lambda)\hat{j}+(1+5\lambda)\hat{k}$ .....(1)

Let P be the point where the line AB crosses the XY plane. Then the position vector $\vec{r}$ of the point P is the form $x\hat{i}+y\hat{j}$

Then, from (1), we have,

$x\hat{i}+y\hat{j}=(3+2\lambda)\,\hat{i}+(4-3\lambda)\,\hat{j}+(1+5\lambda)\,\hat{k}$

$\Rightarrow x=3+2\lambda, y=4-3\lambda\ , 1+5\lambda=0$

Now, $1+5\lambda$ gives, $\lambda=-\frac{1}{5}$

$\therefore x=3+2(\frac{-1}{5})$ and $y=4-3(\frac{-1}{5})$

$\Rightarrow x=\frac{13}{5}$ and $y=\frac{23}{5}$

Hence the required point is $\left(\frac{13}{5},\frac{23}{5},0\right)$

35. Let $\mathbf{u}=\sin^{-1}(2x\sqrt{1-x^2})$

Put x = sin θ

$\Rightarrow u=\sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$\Rightarrow u=\sin^{-1}(2\sin\theta\cos\theta)$

$\Rightarrow u=\sin^{-1}(\sin 2\theta)$ ....(i)

and

Let, v = $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$\Rightarrow v=\sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$

$\Rightarrow v=\sec^{-1}\left(\frac{1}{\cos\theta}\right)$

$\Rightarrow v=\sec^{-1}(\sec\theta)$

$\Rightarrow v=\cos^{-1}\left(\frac{1}{\frac{1}{\cos\theta}}\right)\left[\text{since}, \sec^{-1}x=\cos^{-1}\left(\frac{1}{x}\right)\right]$

$\Rightarrow v=\cos^{-1}(\cos\theta)$ ...(ii)

Here,

$x\in\left(\frac{1}{\sqrt{2}},1\right)$

$\Rightarrow \sin\theta\in\left(\frac{1}{\sqrt{2}},1\right)$

$\Rightarrow \theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

So, from equation (i),

$u=2\theta \quad \left[\text{since}, \sin^{-1}(\sin\theta)=\theta, \text{ if } \theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$

Let u = 2 $\sin^{-1}$x ...[Since, x = sin θ]

Differentiating it with respect to x,

$\frac{du}{dx}=2\left(\frac{1}{\sqrt{1-x^2}}\right)$

$\Rightarrow \frac{du}{dx}=\frac{2}{\sqrt{1-x^2}}$ ....(iii)

And, from equation (ii),

$v=\theta\left[\text{since, } \cos^{-1}(\cos\theta)=\theta, \text{ if } \theta\in[0,\pi]\right]$

$\Rightarrow v=\sin^{-1}x\left[\text{since}, x=\sin\theta\right]$

Differentiating it with respect to x,

$\frac{dv}{dx}=\frac{1}{\sqrt{1-x^2}}$ ...(iv)

dividing equation (iii) by (iv),

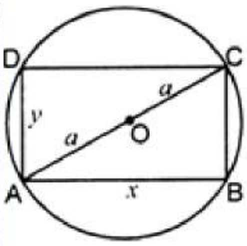
$\frac{du}{dx}=\frac{2}{\sqrt{1-x^2}}\times\frac{\sqrt{1-x^2}}{1}$

$\therefore \frac{du}{dv}=2$.

Section E

36. **Read the text carefully and answer the questions:**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Let ‘y’ be the breadth and ‘x’ be the length of rectangle and ‘a’ is radius of given circle.

From fig $4a^2 = x^2 + y^2$

$\Rightarrow y^2 = 4a^2 - x^2$

$\Rightarrow y = \sqrt{4a^2 - x^2}$

Perimeter (P) = 2x + 2y = $2\left(x+\sqrt{4a^2-x^2}\right)$

(ii) We know that P = $2\left(x+\sqrt{4a^2-x^2}\right)$

Critical points to maximize perimeter $\frac{dP}{dx}$ = 0

$\Rightarrow \frac{dp}{dx}=2\left(1+\frac{1}{2\sqrt{4a^2-x^2}}(-2x)\right)$ = 0

$2\left(\frac{\sqrt{4a^2-x^2}-x}{\sqrt{4a^2-x^2}}\right)$ = 0

$\Rightarrow \sqrt{4a^2-x^2}$ = x

$\Rightarrow 4a^2 - x^2 = x^2$

$\Rightarrow 2a^2 = x^2$

$\Rightarrow x = \pm\sqrt{2a}$

when x = $\sqrt{2a}$, y = $\sqrt{2a}$

when x = $-\sqrt{2a}$ not possible as 'x' is length critical point is $(\sqrt{2a}, \sqrt{2a})$

(iii) $\frac{dp}{dx} = 2\left(1 + \frac{1}{2\sqrt{4a^2-x^2}}(-2x)\right)$

$$\frac{d^2P}{dx^2} = -2\left(\frac{\sqrt{4a^2-x^2}-(x)\left(\frac{-2x}{2\sqrt{4^2-x^2}}\right)}{(4a^2-x^2)}\right)$$

$= -2\left(\frac{(4a^2-x^2)+x^2}{(4a^2-x^2)^{3/2}}\right)$

$\Rightarrow \left.\frac{d^2P}{dx^2}\right]_{x=a\sqrt{2}} = -2\left(\frac{4a^2}{(4a^2-2a^2)^{3/2}}\right) = \frac{-2}{(2\sqrt{2})a} < 0$

Perimeter is maximum at a critical point.

OR

From the above results know that x = y = $\sqrt{2}a$

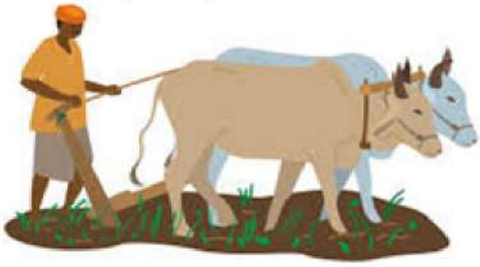
a = radius

Here, x = y = $10\sqrt{2}$

Perimeter = P = 4 $\times$ side = $40\sqrt{2}$ cm

37. **Read the text carefully and answer the questions:**

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

October sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

(i) A + B = $\begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} + \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$

$= \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}$

The combined sales of Masoor in September and October, for farmer Girish ₹40000.

(ii) A + B = $\begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} + \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$

$= \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}$

The combined sales of Urad in September and October, for farmer Ankit is ₹15000.

(iii) A - B = $\begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} - \begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$

$= \begin{bmatrix} 10,000-5000 & 20,000-10,000 & 30,000-6000 \\ 50,000-20,000 & 30,000-10,000 & 10,000-10,000 \end{bmatrix}$

A - B = $\begin{bmatrix} 5000 & 10,000 & 24,000 \\ 30,000 & 20,000 & 0 \end{bmatrix} \begin{matrix} \text{Ankit} \\ \text{Girish} \end{matrix}$

OR

Profit = 2% $\times$ sales on october

= $\frac{2}{100} \times B$

= 0.02 $\times \begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$

= $\begin{bmatrix} 0.02 \times 5000 & 0.02 \times 10,000 & 0.02 \times 6000 \\ 0.02 \times 20,000 & 0.02 \times 10,000 & 0.02 \times 10,000 \end{bmatrix}$

= $\begin{bmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{bmatrix} \begin{matrix} \text{Ankit} \\ \text{Girish} \end{matrix}$

38. **Read the text carefully and answer the questions:**

In a bilateral cricket series between India and South Africa, the probability that India wins the first match is 0.6. If India wins any match, then the probability that it wins the next match is 0.4, otherwise, the probability is 0.3. Also, it is given that there is no tie in any match.

(i) It is given that if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 0.3

(ii) It is given that, if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 1 - 0.3 = 0.7