CBSE SAMPLE PAPER - 04

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. The direction ratios of two lines are 3, 2, -6 and 1, 2, 2 respectively. The acute angle between these lines is **[1]**

a) $\cos^{-1}\left(\frac{5}{18}\right)$
b) $\cos^{-1}\left(\frac{8}{21}\right)$
c) $\cos^{-1}\left(\frac{5}{21}\right)$
d) $\cos^{-1}\left(\frac{3}{20}\right)$

2. $\int e^x \left\{\tan^{-1} x + \frac{1}{(1+x^2)}\right\}$ dx = ? **[1]**

a) $e^x \tan^{-1} x + C$
b) $e^x \cdot \frac{1}{(1+x^2)} + C$
c) None of these
d) $-e^x \cot^{-1} x + C$

3. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, then which one of the following is correct? **[1]**

a) $\vec{a}$ is parallel to $\vec{b}$
b) $\vec{a} = 0$ or $\vec{b} = 0$
c) $\vec{a}$ is perpendicular to $\vec{b}$
d) None of these

4. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is **[1]**

a) $\frac{4(4\pi-\sqrt{3})}{3}$
b) $\frac{4(4\pi+\sqrt{3})}{3}$
c) $\frac{4(8\pi-\sqrt{3})}{3}$
d) $\frac{4(8\pi+\sqrt{3})}{3}$

5. The area bounded by y =| sinx |, the x – axis and the line $|x| = \pi$ is **[1]**

a) none of these
b) 4 sq. units
c) 6 sq. units
d) 2 sq. units

6. Let A and B be two events such that P(A) = 0.6, P(B) = 0.2, and P(A | B) = 0.5. Then P (A′|B′) equals **[1]**

a) $\frac{3}{10}$

b) $\frac{6}{7}$

c) $\frac{3}{8}$

d) $\frac{1}{10}$

7. Let $\mathrm{P(A)} = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $\mathrm{P(A \cap B)} = \frac{4}{13}$. Then $P\left(\frac{A'}{B}\right)$ is equal to **[1]**

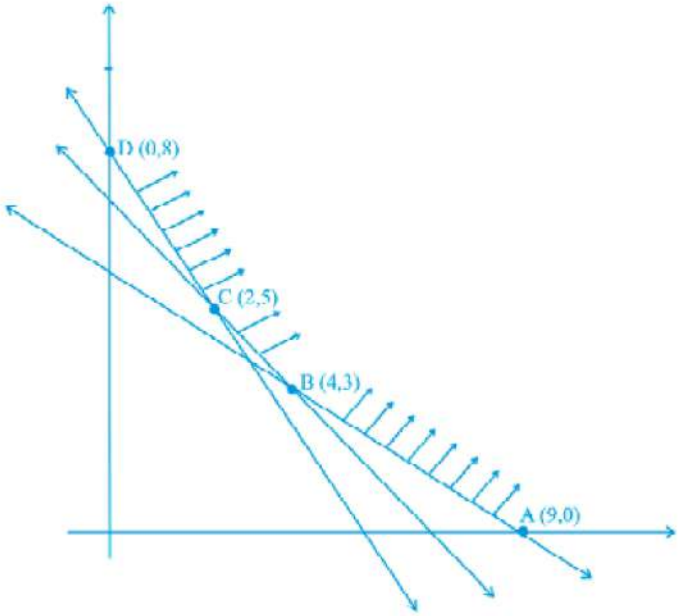
a) $\frac{4}{9}$

b) $\frac{4}{13}$

c) $\frac{5}{9}$

d) $\frac{6}{13}$

8. Feasible region (shaded) for a LPP is shown in the Figure. Minimum of Z = 4x + 3y occurs at the point **[1]**



a) (4, 3)

b) (9, 0)

c) (0, 8)

d) (2, 5)

9. The magnitude of the vector $6\hat{i} + 2\hat{j} + 3\hat{k}$ is **[1]**

a) 7

b) 5

c) 12

d) 1

10. Find a particular solution of $\frac{dy}{dx} = y \tan x$; y =1 when x = 0 **[1]**

a) y = tan x

b) y = sec x

c) y = sin x

d) y = cos x

11. Solution of the differential equation $\frac{dy}{dx} + \frac{y}{x}$ = sin x is: **[1]**

a) x (y + cos x) = cos x + c

b) x (y + cos x) = sin x + c

c) x (y - cos x) = sin x + c

d) xy cos x = sin x + c

12. If f(x) is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5} t^5$ then $f\left(\frac{4}{25}\right)$ equals **[1]**

a) $\frac{2}{5}$

b) $-\frac{5}{2}$

c) $\frac{5}{2}$

d) 1

13. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right)$ + x = 0 is **[1]**

a) $y = -e^{x} + C$

b) $y = e^{-x} + C$

c) $y = -e^{-x} + C$

d) $y = e^{x} + C$

14. If $y = \tan^{-1}x$ and $z = \cot^{-1}x$ then $\frac{dy}{dz}$ is equal to **[1]**

a) 1

b) $\frac{\pi}{2}$

c) – 1

d) None of these

15. Let A be a non-singular square matrix of order 3×3. Then |adj A| is equal to **[1]**

a) | A |

b) 3| A |

c) $| A |^3$

d) $| A |^2$

16. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$ are: **[1]**

a) 1, 4

b) 3, 4

c) 2, 4

d) 3, 2

17. The greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ are respectively **[1]**

a) $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$

b) $\frac{\pi}{2}$ and $\frac{-\pi}{2}$

c) $\frac{\pi^2}{4}$ and 0

d) $\frac{\pi^2}{4}$ and $\frac{-\pi^2}{4}$

18. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k = ? **[1]**

a) $\frac{-10}{7}$

b) $\frac{5}{7}$

c) $\frac{-5}{7}$

d) $\frac{10}{7}$

19. **Assertion (A):** The constraints $-x_1 + x_2 \leq 1$, $-x_1 + 3x_2 \geq 9$ and $x_1, x_2 \geq 0$ defines an unbounded feasible space. **[1]**

Reason (R): The maximum value of Z = 4x + 2y subject to the constraints $2x + 3y \leq 18$, $x + y \geq 10$ and $x, y \geq 0$ is 5.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If f(x) = $\frac{\sin(ax+b)}{\cos(cx+d)}$, then f'(x) = a cos(ax + b) sec(cx + d) + c sin(ax + b) tan(cx + d) sec(cx + d) **[1]**

Reason (R): If f(x) = $\frac{u}{v}$, then f'(x) = $\frac{vu' - uv'}{v^2}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve the differential equation: cosec x log y$\frac{dy}{dx}$ + $x^2 y^2$ = 0 **[2]**

22. Find the value of the constant k so that the given function is continuous at the indicated point: **[2]**

$$f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5}, x \neq 5 \\ k, x = 5 \end{cases} \text{ at } x = 5$$

23. Find the shortest distance between the lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$ **[2]**

OR

If a line has direction ratios $-18, 12, -4$ then what are its direction cosines?

24. Let E_1 and E_2 be two events such that $P(E_1) = 0.3$, $P(E_1 \cup E_2) = 0.4$ and $P(E_2) = x$. Find the value of x such that **[2]**

E_1 and E_2 are independent.

25. Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$. **[2]**

Section C

26. Solve the following LFP graphically: **[3]**

Maximize Z = 20x + 10y

Subject to the following constraints

$x + 2y \leq 28$

$3x + y \leq 24$

$x \geq 2$

$x, y \geq 0$

27. Find the area of the region in the first quadrant enclosed by the x-axis, the line y = x, and the circle $x^2 + y^2 = 32$ **[3]**

OR

Find the area of the region bounded by $y=\sqrt{x}$ and y = x.

28. Evaluate: $\int \frac{\log x}{(x+1)^2} dx$ **[3]**

OR

Evaluate: $\int_0^{\pi/2} \frac{\sin^2 x}{1+\sin x \cos x} dx$

29. Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Also, find the length of the perpendicular. **[3]**

OR

Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r}=(-\hat{i}+3\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}-\hat{k})$. Also, find the image of P in this line.

30. Evaluate the area bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ above the x-axis. **[3]**

31. If $y = \frac{(\cos x-\sin x)}{(\cos x+\sin x)}$, prove that $\frac{dy}{dx} + y^2 + 1 = 0$. **[3]**

Section D

32. If $\vec{a},\vec{b},\vec{c}$ are unit vector such that $\vec{a}+\vec{b}+\vec{c}=0$ find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ **[5]**

33. Solve by matrix method **[5]**

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

OR

If A= $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, then find A^{-1} and

hence solve the system of equations

$x + 2y + z = 4$

$-x + y + z = 0$

and $x - 3y + z = 4$.

34. Show that the function f : R $\rightarrow$ R defined by $f(x)=\frac{x}{x^2+1}$, $\forall x \in R$, is neither one-one nor onto. **[5]**

OR

Let A = R - {3} and B = R - {1}. Consider the function f : A $\rightarrow$ B defined by $f(x)=\left(\frac{x-2}{x-3}\right)$ Is f one-one and onto? Justify your answer.

35. Evaluate: $\int \frac{1}{13+3\cos x+4\sin x} dx$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Find the volume of the open box formed by folding up the cutting each corner with x cm.

(ii) Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(iii) Verify that volume of the box is maximum at x = 3 cm by second derivative test?

OR

Find the maximum volume of the box.

37. **Read the text carefully and answer the questions:** **[4]**

The nut and bolt manufacturing business has gained popularity due to the rapid Industrialization and introduction of the Capital-Intensive Techniques in the Industries that are used as the Industrial fasteners to connect various machines and structures. Mr. Suresh is in Manufacturing business of Nuts and bolts. He produces three types of bolts, x, y, and z which he sells in two markets. Annual sales (in ₹) indicated below:

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

(i) If unit sales prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00 respectively, then find the total revenue collected from Market-I &II.

(ii) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively, then find the cost price in Market I and Market II.

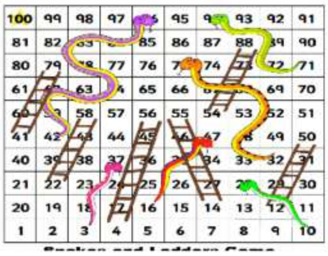
(iii) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively, then find gross profit from both the markets.

OR

If matrix A = $[a_{ij}]_{2\times2}$ where $a_{ij} = 1$, if $i \neq j$ and $a_{ij} = 0$, if i = j then find A^2.

38. **Read the text carefully and answer the questions:** **[4]**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.

Aditya rolled down both black and red die together.

First die is black and second is red.

(i) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(ii) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution

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Class 12 - Mathematics

Section A

1. **(c)** $\cos^{-1}\left(\frac{5}{21}\right)$

Explanation: Direction ratios are given implies that we can write the parallel vector towards that line, lets consider first parallel vector to be $\vec{a}=3\hat{i}+2\hat{j}-6\hat{k}$ and second parallel vector be $\overrightarrow{\mathrm{b}}=\hat{\mathrm{i}}+2\hat{\mathrm{j}}+2\hat{\mathrm{k}}$

For the angle, we can use the formula $\cos\alpha=\frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}|=\sqrt{3^2+2^2+(-6)^2}$

= 7

$|\overrightarrow{\mathrm{b}}|=\sqrt{1+2^2+2^2}$

= 3

$\Rightarrow\cos\alpha=\frac{(3\hat{\imath}+2\hat{\jmath}-6\hat{\mathrm{k}})\cdot(\hat{\mathrm{i}}+2\hat{\mathrm{j}}+2\hat{\mathrm{k}})}{7\times3}$

$\Rightarrow\cos\alpha=\frac{3+4-12}{21}$

$\Rightarrow\cos\alpha=\frac{-5}{21}$

$\therefore\alpha=\cos^{-1}\left(-\frac{5}{21}\right)$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant.

$\alpha=\cos^{-1}\left(\frac{5}{21}\right)$

2. **(a)** $e^x\tan^{-1}x + C$

Explanation: I = $\int e^x\{f(x)+f'(x)\}dx$, where $f(x)=\tan^{-1}x$

$= e^x f(x) + C = e^x\tan^{-1}x + C$

3. **(b)** $\vec{a}=0$ or $\vec{b}=0$

Explanation: Given that, $\vec{a}\cdot\vec{b}=0$,

i.e. $\vec{a}$ and $\vec{b}$ are perpendicular to each other and $\vec{a}\times\vec{b}=0$

i.e. $\vec{a}$ and $\vec{b}$ are parallel to each other. So, both conditions are possible iff $\vec{a}=0$ and $\vec{b}=0$

4. **(c)** $\frac{4(8\pi-\sqrt{3})}{3}$

Explanation: Required area :

$4\int_0^4\sqrt{16-x^2}dx - \int_0^2\sqrt{6x}dx+\int_2^4\sqrt{(16-x^2)}dx = 4\left[\frac{x\sqrt{16-x^2}}{2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4-2\sqrt{6}\left[\frac{x^{3/2}}{3/2}\right]_0^2-2\left[\frac{x\sqrt{16-x^2}}{2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4 =$

$\frac{4}{3}(8\pi-\sqrt{3})$

5. **(b)** 4 sq. units

Explanation: Required area :

$=2\int_0^{\pi}\sin x dx=2[-\cos x]_0^{\pi}=2[1+1]=4 sq. units$

6. **(c)** $\frac{3}{8}$

Explanation: P (A∩B) = P (A | B) P (B)

= 0.5 × 0.2 = 0.1

$P(A'|B')=\frac{P(A'\cap B')}{P(B')}=\frac{P[(A\cup B)']}{1-P(B)}=\frac{1-P(A\cup B)}{1-P(B)}$

$=\frac{1-\mathrm{P(A)}-\mathrm{P(B)}+\mathrm{P(A\cap B)}}{1-0.2}=\frac{3}{8}$

7. **(c)** $\frac{5}{9}$

Explanation: Here, $P(A)=\frac{7}{13}, P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$

$\because P\left(A'|B\right)=\frac{P(A'\cap B)}{P(B)}=\frac{P(B)-P(A\cap B)}{P(B)}$

$=\frac{\frac{9}{13}-\frac{4}{13}}{\frac{9}{13}}=\frac{\frac{5}{13}}{\frac{9}{13}}=\frac{5}{9}$

8. **(d)** (2, 5)

Explanation: Z=4x+3y

1. (0,8)=24

2.(2,5)=8+15=23

3.(4,3)=16+9=25

4. (9,0)=36+0=36

The correct answer is (2, 5) as it gives the minimum value.

9. **(a)** 7

Explanation: 7

10. **(b)** y = sec x

Explanation: $\frac{dy}{y}=tanxdx$

$\int\frac{dy}{y}=\int tanxdx$

$log|y|=log|secx|+logc$

$log|y|=log|csecx|$

$y=csecx$

here y=1 and x=0 gives $1=csec0$

hence c = 1

$\therefore y=secx$

11. **(b)** x (y + cos x) = sin x + c

Explanation: We have, $\frac{dy}{dx}+\frac{1}{x}y$ = sin x

Which is linear differential equation.

Here, P = $\frac{1}{x}$ and Q = sin x

$\therefore$ I.F. = $e^{\int\frac{1}{x}dx}=e^{\log x}=x$

$\therefore$ The general solution is

$y\cdot x=\int x\cdot\sin x dx+C$

= - x cos x - $\int-\cos x dx$

= -x cos x + sin x

$\Rightarrow$ x(y + cos x) = sin x + C

12. **(a)** $\frac{2}{5}$

Explanation: Here, $\int_0^{t^2}$ xf(x) dx = $\frac{2}{5}$ t^5

Using Newton Leibnitz's formula, differentiating both sides, we get

t^2 {f(t^2)} $\left\{\frac{d}{dt}(t^2)\right\}-0\cdot f(0)\left\{\frac{d}{dt}(0)\right\}$ = $2t^4$

$\Rightarrow t^2f(t^2)\,2t=2t^4\Rightarrow f(t^2)=t$

$\therefore\quad f\left(\frac{4}{25}\right)=-\frac{2}{5}$ [putting t = $\frac{2}{5}$]

$\Rightarrow\quad f\left(\frac{4}{25}\right)=\frac{2}{5}$

13. **(c)** y = $-e^{-x}$ + C

Explanation: Given differential equation is

$\log\left(\frac{dy}{dx}\right)+x=0\Rightarrow\log\left(\frac{dy}{dx}\right)=-x$

$\Rightarrow\frac{dy}{dx}=e^{-x}\Rightarrow\int dy=\int e^{-x}\cdot dx$

On integrating both sides, we get y

y = $-e^{-x}$ + C

which is the required general solution.

14. **(c)** – 1

Explanation: $\frac{dy}{dz} = \frac{\frac{d}{dx}(\tan^{-1}x)}{\frac{d}{dx}(\cot^{-1}x)} = \frac{\frac{1}{1+x^2}}{-\frac{1}{1+x^2}} = -1$

15. **(d)** | A |^2

Explanation: For a square matrix of order n×n,

We know that $A.\,adjA = |A|I$

Here, n=3

$\therefore\ |A.\,adjA| = |A|^n$

$|adjA| = |A|^{n-1}$

So, $|AdjA| = |A|^{3-1} = |A|^2$

16. **(d)** 3, 2

Explanation: We have, $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$

$\therefore$ Order = 3 and degree = 2

17. **(a)** $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$

Explanation: We have

$(\sin^{-1}x)^2 + (\cos^{-1}x)^2 = (\sin^{-1}x + \cos^{-1}x)^2 - 2\sin^{-1}x\cos^{-1}x$

$= \frac{\pi^2}{4} - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= \frac{\pi^2}{4} - \pi\sin^{-1}x + 2(\sin^{-1}x)^2$

$= 2\left[\left(\sin^{-1}x\right)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right]$

$= 2\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$

Thus, the least value is $2\left(\frac{\pi^2}{16}\right)$ i.e. $\frac{\pi^2}{8}$ and the Greatest value is $2\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$ i.e. $\frac{5\pi^2}{4}$.

18. **(a)** $\frac{-10}{7}$

Explanation: If the lines are perpendicular to each other then the angle between these lines will be $\frac{\pi}{2}$, then the cosine will be 0

$\vec{a} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$

$\Rightarrow |\vec{a}| = \sqrt{3^2 + (2k)^2 + 2^2}$

$= \sqrt{13 + 4k^2}$

$\vec{b} = 3k\hat{i} + \hat{j} - 5\hat{k}$

$\Rightarrow |\vec{b}| = \sqrt{(3k)^2 + 1 + 5^2}$

$= \sqrt{9k^2 + 26}$

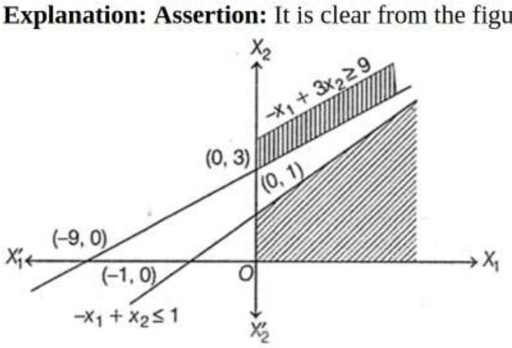
$\cos\left(\frac{\pi}{2}\right) = \frac{(3k\hat{i} + \hat{j} - 5\hat{k})\cdot(-3\hat{i} + 2k\hat{j} + 2\hat{k})}{\sqrt{13+4k^2}\times\sqrt{9k^2+26}}$

$0 = \frac{-9k + 2k - 10}{\sqrt{13+4k^2}\times\sqrt{9k^2+26}}$

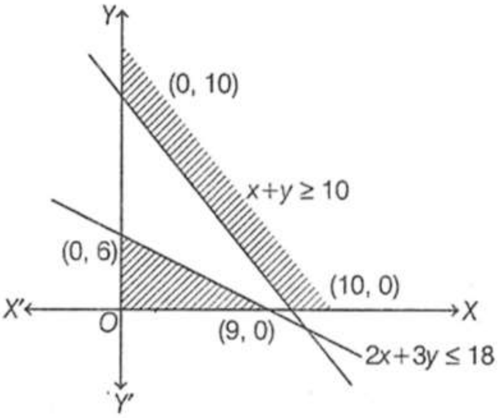
$\Rightarrow k = -\frac{10}{7}$

19. **(c)** A is true but R is false.

Explanation: **Assertion:** It is clear from the figure that feasible space (shaded portion) is unbounded.



Reason: From the figure, it is clear that there is no common area. So, we cannot find maximum value of Z.



Hence, reason is not true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Let y = $\frac{\sin(ax+b)}{\cos(cx+d)}$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin(ax+b)}{\cos(cx+d)}\right)$

$= \frac{\cos(cx+d)\frac{d}{dx}\{\sin(ax+b)\} - \sin(ax+b)\frac{d}{dx}\cos(cx+d)}{[\cos(cx+d)]^2}$ [by quotient rule]

$= \frac{cos(cx+d)cos(ax+b)(a+0)+\sin(ax+b)\sin(cx+d)(c+0)}{\cos^2(cx+d)}$

[by chain rule,

$\frac{d}{dx}$ sin(ax + b) = cos(ax + b) $\frac{d}{dx}$(ax + b)

= cos (ax + b) × (a × 1 + 0)

$\frac{d}{dx}$ cos (cx + d) = -sin(cx + d) $\frac{d}{dx}$ (cx + d)

= -sin (cx + d) × (c × 1 + 0)]

$= \frac{acos(cx+d)cos(ax+b)+csin(ax+b)sin(cx+d)}{cos^2(cx+d)}$

$= \frac{a\cos(cx+d)\cos(ax+b)}{\cos^2(cx+d)} + \frac{c\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$

$= \frac{a\cos(ax+b)}{\cos(cx+d)} + \frac{c\sin(ax+b)\sin(cx+d)}{\cos(cx+d)\cos(cx+d)}$

= a cos (ax + b) sec (cx + d) + c sin (ax + b) tan (cx + d) sec (cx + d)

Section B

21. We have, cosec x log y$\frac{dy}{dx}$ + x^2 y^2 = 0

⇒ cosec x log y$\frac{dy}{dx}$ = -x^2 y^2

⇒ $\frac{1}{y^2}\log ydy = -\frac{x^2}{cosecx}dx$

⇒ $\frac{1}{y^2}$ log y dy = -x^2 sin x dx

⇒ $\int \frac{1}{y^2}$ log y dy = $-\int$ x^2 sin x dx

⇒ $-\frac{\log y}{y} + \int \frac{1}{y} \times \frac{1}{y} = -\left[-x^2\cos x + \int 2x\cos x dx\right] + C$

⇒ $-\frac{\log y}{y} - \frac{1}{y} = -\left[-x^2\cos x + 2x\sin x - 2\int \sin x dx\right] + C$

⇒ $-\left(\frac{1+\log y}{y}\right)$ = - [-x^2 cos x + 2x sin x + 2 cos x dx] + C

⇒ $-\left(\frac{1+\log y}{y}\right)$ - x^2 cos x + 2 (x sin x + cos x) = C

Hence It is the required solution.

22. Given: $f(x) = \begin{cases} \dfrac{x^2-25}{x-5}, x \neq 5 \\ k, x = 5 \end{cases}$

$\Rightarrow f(x) = \begin{cases} \dfrac{x^2-25}{x-5}, x \neq 5 \\ k, x = 5 \end{cases}$

$\Rightarrow f(x) = \begin{cases} \dfrac{x^2-25}{x-5}, x \neq 5 \\ k, x = 5 \end{cases}$

If f(x) is continuous at x = 5, then

$\lim_{x \to 5} f(x) = f(5)$

$\Rightarrow \lim_{x \to 5}(x+5) = k$

$\Rightarrow$ 5 + 5 = k

Hence, k = 10

23. Comparing the given equations with $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, we obtain

(x_1 = - 3, y_1 = 6, z_1 = 0), (x_2 = -2, y_2 = 0, z_2 = 7).

(a_1, = -4, b_1 = 3, c_1 = 2) and (a_2 = -4, b_2 = 1, c_2 = 1)

Now, we have D = $(a_2b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2$

= $(-4 + 12)^2 + (3 - 2)^2 + (-8 + 4)^2$

= (64 + 1 + 16) = 81

$\therefore \text{SD} = \frac{1}{\sqrt{D}} \cdot \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$= \frac{1}{\sqrt{81}} \cdot \begin{vmatrix} -2+3 & 0-6 & 7-0 \\ -4 & 3 & 2 \\ -4 & 1 & 1 \end{vmatrix} = \frac{1}{9} \cdot \begin{vmatrix} 1 & -6 & 7 \\ -4 & 3 & 2 \\ -4 & 1 & 1 \end{vmatrix}$

$= \frac{1}{9} \cdot \{1 \cdot (3-2) + 6 \cdot (-4+8) + 7 \cdot (-4+12)\}$

$= \frac{81}{9}$ = 9 units

Therefore, the shortest distance between the given lines is 9 units.

OR

We know that if a,b,c are direction ratios of a line, then direction cosines of the line are:

$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$ ...(i)

Here direction ratios of the line are $-18, 12, -4$

Putting the values in eq. (i),

$= \frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$

$= \frac{-18}{\sqrt{324+144+16}}, \frac{12}{\sqrt{324+144+16}}, \frac{-4}{\sqrt{324+144+16}}$

$= \frac{-18}{\sqrt{484}}, \frac{12}{\sqrt{484}}, \frac{-4}{\sqrt{484}}$

$\Rightarrow \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$

$\Rightarrow \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

Hence, direction cosines of required line are $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$.

24. Let E_1 and E_2 be two independent events. Then, $P(E_1 \cap E_2) = P(E_1) \times P(E_2) = 0.3 \times x = 0.3x$

$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow$ 0.4 + 0.3 + x - 0.3x

$\Rightarrow$ 0.7x = 0.1

$\Rightarrow x = \frac{0.1}{0.7} = \frac{1}{7}$

25. We have, $\cos^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\cos\frac{\pi}{3}\right)$

$= \frac{\pi}{3} \left[\because \frac{\pi}{3} \in [0, \pi]\right]$

Also $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$= \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

$= -\frac{\pi}{6} \left[\because -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

Section C

26. Subject to the constraints are

$x + 2y \leq 28$

$3x + y \leq 24$

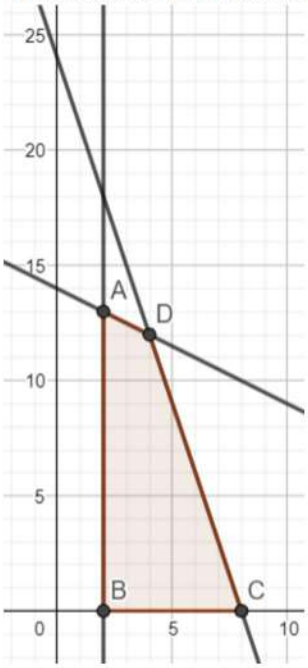
$x \geq 2$

and the non-negative restrictions $x, y \geq 0$

Converting the given inequations into equations, we get

x + 2y = 28, 3x + y = 24, x = 2, x = 0 and y = 0

These lines are drawn on the graph and the shaded region ABCD represents the feasible region of the given LPP.



It can be observed that the feasible region is bounded. The coordinates of the corner points of the feasible region are A(2, 13), B(2, 0), C(4, 12) and D(8, 0)

The values of the objective function, Z at these corner points are given in the following table:

Corner Point Value of the Objective Function Z = 20x + 10y

A(2, 13) : $Z = 20 \times 2 + 10 \times 13 = 170$

B(2, 0) : $Z = 20 \times 2 + 10 \times 0 = 40$

C(8, 0) : $Z = 20 \times 8 + 10 \times 0 = 160$

D(4, 12) : $Z = 20 \times 4 + 10 \times 12 = 200$

From the table, Z is maximum at x = 4 and y = 12 and the maximum value of objective function Z is 200.

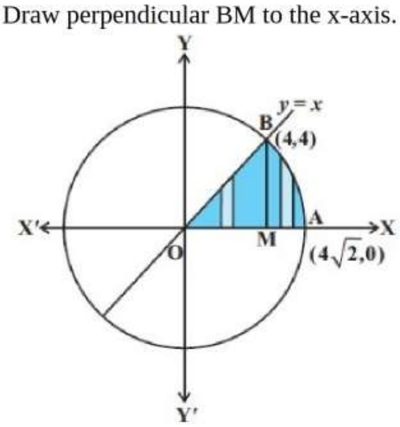
27. The given equations are

y = x ... (1)

and $x^2 + y^2 = 32$ ... (2)

Solving (1) and (2), we find that the line and the circle meet at B(4, 4) in the first quadrant.

Draw perpendicular BM to the x-axis.



Thus, the required area = area of the region OBMO + area of the region BMAB.

Now, the area of the region OBMO

$= \int_0^4 y dx = \int_0^4 x dx$

$= \frac{1}{2}\left[x^2\right]_0^4 = 8$ .....(3)

Again, the area of the region BMAB

$= \int_4^{4\sqrt{2}} y dx = \int_4^{4\sqrt{2}} \sqrt{32 - x^2} dx$

$= \left[\frac{1}{2} x \sqrt{32 - x^2} + \frac{1}{2} \times 32 \times \sin^{-1} \frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$

$= \left(\frac{1}{2} 4\sqrt{2} \times 0 + \frac{1}{2} \times 32 \times \sin^{-1} 1\right) - \left(\frac{4}{3}\sqrt{32 - 16} + \frac{1}{2} \times 32 \times \sin^{-1} \frac{1}{\sqrt{2}}\right)$

$= 8\pi - (8 + 4\pi) = 4\pi - 8$ ..(4)

Adding (3) and (4), we get, the required area = 4π. Which is the required solution.

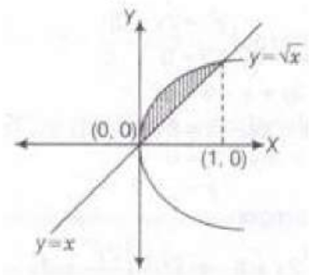
OR

Given equation of curves are $y = \sqrt{x}$ and y = x

$\Rightarrow x = \sqrt{x} \Rightarrow x^2 = x$

$\Rightarrow x^2 - x = 0 \Rightarrow x(x - 1) = 0$

$\Rightarrow x = 0, 1$



$\therefore$ The required area of the shaded region, $A = \int_0^1 (\sqrt{x}) dx - \int_0^1 x dx$

$= \left[2.\frac{x^{3/2}}{3}\right]_0^1 - \left[\frac{x^2}{2}\right]_0^1$

$= \frac{2}{3}.1 - \frac{1}{2} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ sq units.

28. Let the given integral be,

I = $\int \frac{\log x}{(x+1)^2} dx$

Now solving by parts.

Let the first function be (log x) and second function be $\frac{1}{(x+1)^2}$

First we find the integral of the second function, i.e., $\int \frac{1}{(x+1)^2}$

Put t = (x + 1). Then dt = dx

Therefore,

$\int \frac{1}{(x+1)^2} dx = \int$ t^{-2} dt

$= -\frac{1}{t}$

$= -\frac{1}{1+x}$

Hence, using integration by parts, we get

$\int \frac{\log x}{(x+1)^2} dx$ = (log x) $\int \frac{1}{(x+1)^2} dx - \int \left[\left(\frac{d(\log x)}{dx}\right) \int \frac{1}{(x+1)^2} dx\right] dx$

= (log x) $\left(-\frac{1}{1+x}\right) - \int \left(\frac{1}{x}\right)\left(-\frac{1}{1+x}\right)$ dx

$= -\frac{\log x}{1+x} + \int \left(\frac{1}{x^2+x}\right) dx$

$= -\frac{\log x}{1+x} + \int \frac{1}{x^2+x+\frac{1}{4}-\frac{1}{4}} dx$

$= -\frac{\log x}{1+x} + \int \frac{1}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$

$= -\frac{\log x}{1+x} + \frac{1}{2 \times \frac{1}{2}} \log\left|\frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}+\frac{1}{2}}\right| + c$

$= -\frac{\log x}{1+x} + \log\left|\frac{x}{x+1}\right| + c$

Hence, $\int \frac{\log x}{(x+1)^2} dx = -\frac{\log x}{1+x} + \log\left|\frac{x}{x+1}\right| + c$

OR

Let the given integral be,

$I = \int_0^{\pi/2} \frac{\sin^2 x}{1+\sin x \cos x} dx$ ...(i)

$I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)} dx$ $\left[\text{Using} : \int_0^a f(x)dx = \int_0^a f(a-x)\right]$

$\Rightarrow \quad I=\int_0^{\pi/2} \frac{\cos^2 x}{1+\cos x \sin x} dx$ ...(ii)

Adding (i) and (ii), we get

$2I=\int_0^{\pi/2} \frac{\sin^2 x+\cos^2 x}{1+\sin x \cos x} dx=\int_0^{\pi/2} \frac{1}{1+\sin x \cos x} dx$

$\Rightarrow \quad 2I=\int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x+\tan x} dx$ [Dividing numerator and denominator by $\cos^2$ x]

$\text{Let } \tan x=t. \text{ Then, } d(\tan x)=dt \Rightarrow \sec^2 x dx=dt$

Also,$x=0 \Rightarrow t=\tan 0=0$ and $x=\frac{\pi}{2} \Rightarrow t=\tan \frac{\pi}{2}=\infty$

$\therefore \quad 2I=\int_0^{\infty} \frac{dt}{1+t^2+t}$

$\Rightarrow \quad 2I=\int_0^{\infty} \frac{dt}{(t+1/2)^2+(\sqrt{3/2})^2} dt=\frac{1}{\sqrt{3}/2}\left[\tan^{-1}\left(\frac{t+1/2}{\sqrt{3/2}}\right)\right]_0^{\infty}$

$\Rightarrow \quad 2I=\frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^{\infty}=\frac{2}{\sqrt{3}}\left[\tan^{-1}\infty \ -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]=\frac{2}{\sqrt{3}}\left[\frac{\pi}{2}-\frac{\pi}{6}\right]=\frac{2\pi}{3\sqrt{3}}$

$\Rightarrow \quad I=\frac{\pi}{3\sqrt{3}}$

29. Let L be the foot of the perpendicular drawn from the point P (0, 2, 3) to the given line. The coordinates of a general point on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$ are given by

$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$

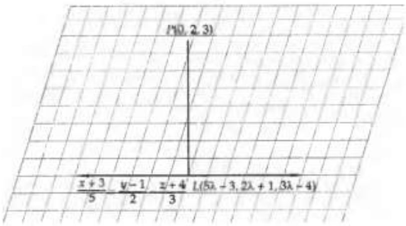
or, x = $5\lambda-3$, y = $2\lambda+1$, z = $3\lambda-4$

Let the coordinates of L be $(5\lambda-3, 2\lambda+1, 3\lambda-4)$.

Therefore, direction ratios of PL are proportional to

$5\lambda-3$ = 0, $2\lambda+1$ = 0, $3\lambda-4$ = 0

i.e. $5\lambda-3$, $2\lambda+1$, $3\lambda-4$



Direction ratios of the given line are proportional to 5,2,3.

But, PL is perpendicular to the given line.

$\therefore 5(5\lambda-3)+2(2\lambda-1)+3(3\lambda-7)=0 \Rightarrow \lambda$ = 1

Putting λ = 1 in $(5\lambda-3, 2\lambda+1, 3\lambda-4)$ the coordinates of L are (2, 3, -1).

$\therefore$ PL = $\sqrt{(2-0)^2+(3-2)^2+(-1-3)^2}$ = $\sqrt{21}$ units

Hence, the length of the perpendicular from P on the given line is PL = $\sqrt{21}$ units.

OR

According to question , the vector equation of the given line is

$\vec{r}=(-\hat{i}+3\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}-\hat{k})$

Clearly, it passes through the point (-1, 3, 1) and it has direction ratios 2, 3, -1.

So, its Cartesian equations are

$\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}=r$ (say)

The general point on this line is (2r -1, 3r + 3, - r +1)

Suppose N be the foot of the perpendicular drawn from the point P(5, 4, 2) on the given line.

Then, this point is N(2r -1, 3r + 3, - r +1) for some fixed value of r. D.r.'s of PN are(2r - 6, 3r -1, - r -1).

Direction ratios of the given line are 2, 3, -1.

Since PN is perpendicular to the given line (i), we have

2(2r - 6) + 3(3r -1) -1 • (-r -1) = 0 ⇒ 14r = 14 ⇒ r = 1

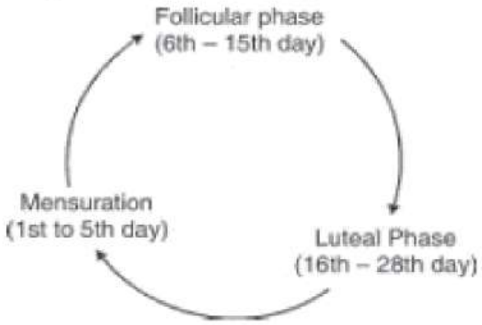
So, the point N is given by N(l, 6, 0).

Hence, the foot of the perpendicular from the given point P(5,4,2) on the given line is N(1, 6,0).

Suppose Q (α, β, γ) be the image of P(5,4, 2) in the given line.

Then, N(1, 6, 0) is the midpoint of PQ

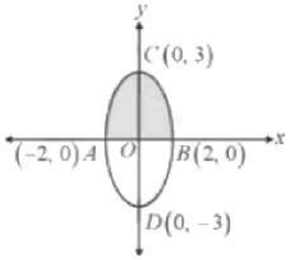
$\therefore \frac{5+\alpha}{2}=1, \frac{4+\beta}{2}=6$ and $\frac{2+y}{2}=0 \Rightarrow \alpha=-3$,$\beta=8$ and $\gamma=-2$



30. The given equation, $\frac{x^2}{4}+\frac{y^2}{9}$ = 1 is a right handed ellipse with its vertex at the origin and the point of intersection $(\pm 2,0)$ and $(0,\pm 3)$. Also, $\frac{x^2}{4}+\frac{y^2}{9}$ = 1 contains only even power of x and y.

so, it is symmetrical about the x-axis, y-axis respectively.

Therefore Required area of shaded region= 2 area of OBC = $2\int\limits_0^2 y dx=2\int\limits_0^2 \frac{3}{2}\sqrt{4-x^2}$



$=3\int\limits_0^2 \sqrt{(2)^2-(x)^2}dx=3\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$=3\left[\left\{\frac{2}{2}\sqrt{4-4}+\frac{4}{2}\sin^{-1}\left(\frac{2}{2}\right)\right\}-\left\{\frac{0}{2}\sqrt{4-0}+2\sin^{-1}(0)\right\}\right]$

= 3 [2 $\sin^{-1}$ (1)] = $3.2\cdot\frac{\pi}{2}$ sq. units = 3πsq. units

31. Given y = $\frac{(\cos x-\sin x)}{(\cos x+\sin x)}$

Dividing numerator and denominator by cos x

y = $\frac{\left(1-\frac{\sin x}{\cos x}\right)}{\left(1+\frac{\sin x}{\cos x}\right)}$

y = $\frac{1-\tan x}{1+\tan x}$

y = tan $\left(\frac{\pi}{4}-x\right)$

Differentiating above equation w.r.t.x,

$\frac{dy}{dx}$ = $\frac{d}{dx}\tan\left(\frac{\pi}{4}-x\right)$

= $\sec^2\left(\frac{\pi}{4}-x\right)\cdot\frac{d}{dx}\left(\frac{\pi}{4}-x\right)$

= $\sec^2\left(\frac{\pi}{4}-x\right)\left(\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{\pi}{4}\right)-\frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{x})\right)$

= $\sec^2\left(x+\frac{\pi}{4}\right)$ (0 - 1)

= - $\sec^2\left(x+\frac{\pi}{4}\right)$

$\therefore \frac{dy}{dx}$ = -$\sec^2\left(x+\frac{\pi}{4}\right)$

Now,

$\frac{dy}{dx}$ + y^2 + 1 = -$\sec^2\left(x+\frac{\pi}{4}\right)$ + $\left(\tan^2\left(\mathrm{x}+\frac{\pi}{4}\right)+1\right)$

= -$\sec^2\left(x+\frac{\pi}{4}\right)$ + $\left(\sec^2\left(x+\frac{\pi}{4}\right)\right)$

= 0

$\therefore \frac{dy}{dx}$ + y^2 + 1 = 0.

Section D

32. $|\vec{a}|=1,|\vec{b}|=1,|\vec{c}|=1$

$\vec{a}+\vec{b}+\vec{c}=0$ (Given)

$\vec{a}.\left(\vec{a}+\vec{b}+\vec{c}\right)=0$

$\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$

$|\vec{a}|^2+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$

$1+\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$

$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-1$ ...(i)

Similarly,

$\vec{b}.\vec{a}+\vec{b}.\vec{c}=-1$ ...(ii)

again

$\vec{c}.\vec{a}+\vec{c}.\vec{b}=-1$ ...(iii)

adding (i), (ii) and (iii)

$2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=-3\left[\because \vec{a}.\vec{b}=\vec{b}.\vec{a}\right]$

$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-\frac{3}{2}$

33. $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$

$|A|=\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{vmatrix}$

$=10\neq 0$

Also, adj $A=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$

$A^{-1}=\frac{1}{|A|}(adJA)$

$=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$

System of equation can be written is

$X=A^{-1}B$

$=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$

$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$

x=2. y = -1. z = 1

OR

Given, A= $\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 1\\ 1 & -3 & 1\end{bmatrix}$

Now, |A| = 1 (1 + 3) - 2 (- 1 -1) + 1 (3-1)

= 4 + 4 + 2=10

$\Rightarrow \quad |A|\neq 0$, hence A^{-1} exists.

Now, cofactors of elements of |A| are,

$A_{11}=(-1)^2\begin{vmatrix}1 & 1\\ -3 & 1\end{vmatrix}=(1+3)=4$

$A_{12}=(-1)^3\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}=-(-1-1)=2$

$A_{13}=(-1)^4\begin{vmatrix}-1 & 1\\ 1 & -3\end{vmatrix}=(3-1)=2$

$A_{21}=(-1)^3\begin{vmatrix}2 & 1\\ -3 & 1\end{vmatrix}=-(2+3)=-5$

$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = (1-1) = 0$

$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} = -1(-3-2) = 5$

$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (2-1) = 1$

$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = -(1+1) = -2$

$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = (1+2) = 3$

$\therefore \operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$

and $A^{-1} = \frac{1}{|A|}\operatorname{adj}(A)$

$\Rightarrow A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$

Given system of equations can be written in matrix from as

$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$

i.e. AX = B

where,

$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix} B = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Clearly, $X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ 8+0+(-8) \\ 8+0+12 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

Therefore,on comparing corresponding elements, we get x = 2, y = 0 and z = 0

34. For $x_1, x_2 \in R$, consider

$f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$

$\Rightarrow x_1x_2^2 + x_1 = x_2x_1^2 + x_2$

$\Rightarrow x_1x_2(x_2 - x_1) = x_2 - x_1$

$\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$

We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$ for instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1)\frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$. Hence f is not one-one. Also, f is not onto for if so then for $1 \in R \ \exists\ x \in R$ such that f(x) = 1 which gives $\frac{x}{x^2+1} = 1$. But there is no such x in the domain R, since the equation $x^2 - x + 1 = 0$ does not give any real value of x.

OR

A = R - {3} and B = R - {1} and $f(x) = \frac{x-2}{x-3}$

Let $x_1, x_2 \in A$, then $f(x_1) = \frac{x_1-2}{x_1-3}$ and $f(x_2) = \frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2 \therefore$ f is one-one function.

Now, $y = \frac{x-2}{x-3}$

$\Rightarrow y(x - 3) = x - 2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y - 1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{3y-2-3y+3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

35. Let the given integral be

$I = \int \frac{1}{13+3\cos x+4\sin x} dx$

Putting $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\sin x = \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

$\therefore I = \int \frac{1}{13+3\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+4\times 2\frac{\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}} dx$

$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{13\left(1+\tan^2\frac{x}{2}\right)+3-3\tan^2\frac{x}{2}+8\tan\left(\frac{x}{2}\right)} dx$

$= \int \frac{\sec^2\frac{x}{2}}{13\tan^2\frac{x}{2}-3\tan^2\frac{x}{2}+16+8\tan\left(\frac{x}{2}\right)} dx$

$= \int \frac{\sec^2\frac{x}{2}}{10\tan^2\left(\frac{x}{2}\right)+8\tan\left(\frac{x}{2}\right)+16} dx$

Let $\tan\left(\frac{x}{2}\right) = t$ then we have

$\Rightarrow \frac{1}{2}\sec^2\left(\frac{x}{2}\right) dx = dt$

$\Rightarrow \sec^2\left(\frac{x}{2}\right) dx = 2dt$

$\therefore I = \int \frac{2dt}{10t^2+8t+16}$

$= \int \frac{dt}{5t^2+4t+8}$

$= \frac{1}{5}\int \frac{dt}{t^2+\frac{4}{5}t+\frac{8}{5}}$

$= \frac{1}{5}\int \frac{dt}{t^2+\frac{4}{5}t+\left(\frac{2}{5}\right)^2-\left(\frac{2}{5}\right)^2+\frac{8}{5}}$

$= \frac{1}{5}\int \frac{dt}{\left(t+\frac{2}{5}\right)^2-\frac{4}{25}+\frac{8}{5}}$

$= \frac{1}{5}\int \frac{dt}{\left(t+\frac{2}{5}\right)^2+\frac{-4+40}{25}}$

$= \frac{1}{5}\int \frac{dt}{\left(t+\frac{2}{5}\right)^2+\left(\frac{6}{5}\right)^2}$

$= \frac{1}{5} \times \frac{5}{6}\tan^{-1}\left(\frac{t+\frac{2}{5}}{\frac{6}{5}}\right) + C$

$= \frac{1}{6}\tan^{-1}\left(\frac{5t+2}{6}\right) + C$

$= \frac{1}{6}\tan^{-1}\left(\frac{5\tan\frac{x}{2}+2}{6}\right) + C$

Section E

36. **Read the text carefully and answer the questions:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Let the side of square to be cut off be 'x' cm. then, the length and the breadth of the box will be (18 - 2x) cm each and the height of the box is 'x' cm.

The volume V(x) of the box is given by $V(x) = x(18 - x)^2$

(ii) $V(x) = x(18 - 2x)^2$

$\frac{dV(x)}{dx} = (18 - 2x)^2 - 4x(18 - 2x)$

For maxima or minima = $\frac{dV(x)}{dx} = 0$

$\Rightarrow (18 - 2x)[18 - 2x - 4x] = 0$

$\Rightarrow x = 9$ or $x = 3$

$\Rightarrow x =$ not possible

$\Rightarrow x = 3$ cm

The side of the square to be cut off so that the volume of the box is maximum is x = 3 cm

(iii) $\frac{dV(x)}{dx} = (18 - 2x)(18 - 6x)$

$\frac{d^2V(x)}{dx^2} = (18 - 6x)(-2) + (18 - 2x)(-6)$

$\Rightarrow \frac{d^2V(x)}{dx^2} = -12[3 - x + 9 - x] = -24(6 - x)$

$\Rightarrow \left.\frac{d^2V(x)}{dx^2}\right]_{x=3} = -72 < 0$

$\Rightarrow$ volume is maximum at x = 3

OR

$V(x) = x(18 - 2x)^2$

When x = 3

$V(3) = 3(18 - 2 \times 3)^2$

$\Rightarrow$ Volume $= 3 \times 12 \times 12 = 432\ cm^3$

37. **Read the text carefully and answer the questions:**

The nut and bolt manufacturing business has gained popularity due to the rapid Industrialization and introduction of the Capital-Intensive Techniques in the Industries that are used as the Industrial fasteners to connect various machines and structures. Mr. Suresh is in Manufacturing business of Nuts and bolts. He produces three types of bolts, x, y, and z which he sells in two markets. Annual sales (in ₹) indicated below:

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

(i) Let A be the 2×3 matrix representing the annual sales of products in two markets.

$$\begin{matrix} & x & y & z & \\ A = & \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} & & & \begin{matrix} Market\ I \\ Market\ II \end{matrix} \end{matrix}$$

Now, revenue = sale price × number of items sold

$$AB = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$$

Therefore, the revenue collected from Market I = ₹46000 and the revenue collected from Market II = ₹53000.

(ii) Let C be the column matrix representing cost price of each unit of products x, y, z.

$$C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$\Rightarrow AC = \begin{bmatrix} 20000 + 2000 + 9000 \\ 12000 + 20000 + 4000 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

Cost price in Market I is ₹31000 and in Market is ₹36000.

(iii)Now, Profit matrix = Revenue matrix - Cost matrix

$\Rightarrow$ AB - AC

$$\Rightarrow \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix} = \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$$

Therefore, the gross profit from both the markets = ₹15000 + ₹17000 = ₹32000

OR

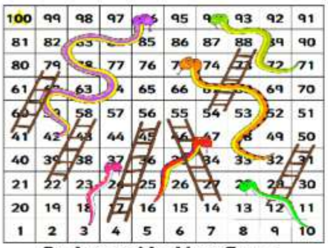
$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\Rightarrow A^2 = I$

38. **Read the text carefully and answer the questions:**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.



Aditya rolled down both black and red die together.

First die is black and second is red.

(i) Let A represents obtaining a sum greater than 9 and B represents black die resulted in a 5.

n(S) = 36

n(A) = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)} = 6

n(B) = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5)} = 6

n(A ∩ B) = {(5, 5), (5, 6)} = 2

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{n(A\cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$$

(ii) Let A represents obtaining a sum 8 and B represents red die resulted in number less than 4.

n(S) = 36

n(A) = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) = 5

n(B) = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)} = 18 n(A ∩ B) = {(5, 3), (6, 2)} = 2

P(A/B) = $\frac{P(A \cap B)}{P(B)}$ = $\frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$ = $\frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$