



- a) 0.5  
c) 0.2
- b) 0.4  
d) 0.3
7. The area bounded by the curves  $y = \sqrt{x}$ ,  $2y + 3 = x$  and the x-axis in the first quadrant is [1]  
a) 36  
b) 18  
c) 9  
d) none of these
8. The angle between the straight lines  $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$  and  $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$  is [1]  
a)  $45^\circ$   
b)  $60^\circ$   
c)  $30^\circ$   
d)  $90^\circ$
9. If  $|\vec{a}| = 3$  and  $-1 \leq k \leq 2$ , then  $|k\vec{a}|$  lies in the interval. [1]  
a) [- 3, 6]  
b) [3, 6]  
c) [0, 6]  
d) [1, 2]
10. The differential equation  $y \frac{dy}{dx} + x = c$  represents: [1]  
a) Family of ellipses  
b) Family of hyperbolas  
c) Family of parabolas  
d) Family of circles
11. If the area cut off from a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex, then k is equal to [1]  
a)  $\frac{2}{3}$   
b) 3  
c)  $\frac{1}{3}$   
d)  $\frac{3}{2}$
12.  $\int \tan^{-1} \left( \frac{3x-x^3}{1-3x^2} \right) dx$  can be evaluated by the substitution [1]  
a)  $x = \cos \theta$   
b)  $x = \sec \theta$   
c)  $x = \sin \theta$   
d)  $x = \tan \theta$
13.  $f(x) = \frac{x}{(x^2+1)}$  is increasing in [1]  
a) None of these  
b)  $(-1, \infty)$   
c)  $(-\infty, -1) \cup (1, \infty)$   
d)  $(-1, 1)$
14. The number of all possible matrices of order  $3 \times 3$  with each entry 0 or 1 is [1]  
a) 81  
b) none of these  
c) 512  
d) 18
15. The system  $AX = B$  of n equations in n unknowns has infinitely many solutions if [1]  
a) if  $\det.A = 0$ ,  $(\text{adj } A)B \neq O$   
b)  $\det. A \neq 0$   
c) if  $\det. A \neq 0$ ,  $(\text{adj}A)B \neq O$   
d) if  $\det. A = 0$ ,  $(\text{adj } A ) B = O$
16. The matrix  $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$  is a singular matrix, if the value of b is [1]  
a) 3  
b) Non-existent  
c) -3  
d) 0

17. Range of  $\sin^{-1}x$  is [1]

- a) None of these  
b)  $[0, \pi]$   
c)  $[\frac{-\pi}{2}, \frac{\pi}{2}]$   
d)  $[0, \frac{\pi}{2}]$

18. The general solution of the DE  $\log\left(\frac{dy}{dx}\right) = (ax + by)$  is [1]

- a)  $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$   
b) none of these  
c)  $e^{ax} - e^{-by} = C$   
d)  $be^{ax} + ae^{by} = C$

19. **Assertion (A):** The absolute maximum value of the function  $2x^3 - 24x$  in the interval  $[1, 3]$  is 89. [1]

**Reason (R):** The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

- a) Both A and R are true and R is the correct explanation of A.  
b) Both A and R are true but R is not the correct explanation of A.  
c) A is true but R is false.  
d) A is false but R is true.

20. **Assertion (A):** Minor of an element of a determinant of order  $n(n \geq 2)$  is a determinant of order  $n$ . [1]

**Reason (R):** If A is an invertible matrix of order 2, then  $\det(A^{-1})$  is equal to  $\frac{1}{|A|}$ .

- a) Both A and R are true and R is the correct explanation of A.  
b) Both A and R are true but R is not the correct explanation of A.  
c) A is true but R is false.  
d) A is false but R is true.

### Section B

21. Find the value of  $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$  [2]

22. Find the general solution of the differential equation:  $x\frac{dy}{dx} - y = x + 1$  [2]

23. For the matrices  $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$  verify that  $(AB)^{-1} = B^{-1}A^{-1}$  [2]

OR

Solve the system of linear equation, using matrix method  $2x + 3y + 3z = 5$ ;  $x - 2y + z = -4$ ;  $3x - y - 2z = 3$

24. Write a unit vector in the direction of the sum of the vectors  $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$ . [2]

25. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once? [2]

### Section C

26. Integrate  $\int x^2 \tan^{-1}x dx$  [3]

27. Find the particular solution of differential equation  $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ ,  $x \neq 0$ , given that  $y = 0$ , when  $x = \frac{\pi}{2}$ . [3]

OR

Solve the differential equation  $x(1 + y^2) dx - y(1 + x^2) dy = 0$ , given that  $y = 0$ , when  $x = 1$

28. Find the value of  $\lambda$  so that the four points A, B, C and D with position vectors  $4\hat{i} + 5\hat{j} + \hat{k}$ ,  $-\hat{j} - \hat{k}$ ,  $3\hat{i} + \lambda\hat{j} + 4\hat{k}$  and  $-4\hat{i} + 4\hat{j} + 4\hat{k}$ , respectively are coplanar. [3]

OR

ABCD is a parallelogram. If L and M are the mid-points of BC and DC respectively, then express  $\vec{AL}$  and  $\vec{AM}$  in terms of  $\vec{AB}$  and  $\vec{AD}$ . Also, prove that  $\vec{AL} + \vec{AM} = \frac{3}{2}\vec{AC}$ .

29. Prove that:  $\int_0^{\pi/2} \frac{\sin^{3/2} x}{(\sin^{3/2} x + \cos^{3/2} x)} dx = \frac{\pi}{4}$  [3]

OR

Evaluate:  $\int \frac{dx}{x(x^4-1)}$ .

30. If  $x = ae^t (\sin t + \cos t)$  and  $y = ae^t (\sin t - \cos t)$ , prove that  $\frac{dy}{dx} = \frac{x+y}{x-y}$ . [3]

31. Using the method of integration, find the area of the region bounded by the lines:  $2x + y = 4$ ,  $3x - 2y = 6$  and  $x - 3y + 5 = 0$ . [3]

### Section D

32. Solve the following linear programming problem graphically. Minimise  $z = 3x + 5y$  subject to the constraints [5]

$x + 2y \geq 10$

$x + y \geq 6$

$3x + y \geq 8$

$x, y \geq 0$ .

33. Let  $A = [-1, 1]$ . Then, discuss whether the following functions defined on  $A$  are one-one, onto or bijective: [5]

i.  $f(x) = \frac{x}{2}$

ii.  $g(x) = |x|$

iii.  $h(x) = x|x|$

iv.  $k(x) = x^2$

OR

Let  $R$  be a relation on  $N \times N$ , defined by  $(a, b) R (c, d) \Leftrightarrow a + d = b + c$  for all  $(a, b), (c, d) \in N \times N$ . Show that  $R$  is an equivalence relation.

34. Find the image of the point  $(5, 9, 3)$  in the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  [5]

OR

Find the coordinates of the point where the line through the points  $A(3, 4, 1)$  and  $B(5, 1, 6)$  crosses the  $XY$ -plane.

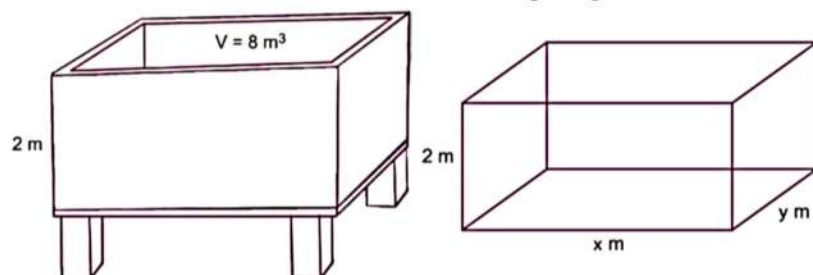
35. Show that the function  $f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$  is continuous but not differentiable at  $x = 0$ , if  $0 < m < 1$ . [5]

1.

### Section E

36. Read the text carefully and answer the questions: [4]

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is  $8 \text{ m}^3$  as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.



(i) Express making cost  $C$  in terms of length of rectangle base.

(ii) If  $x$  and  $y$  represent the length and breadth of its rectangular base, then find the relation between the variables.

(iii) Find the value of  $x$  so that the cost of construction is minimum.

OR

Verify by second derivative test that cost is minimum at a critical point.

37. **Read the text carefully and answer the questions:**

[4]

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.



Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

- Represent the sale of handmade fans, mats and plates by three schools A, B and C and the sale prices (in ₹) of given products per unit, in matrix form.
- Find the funds collected by school A, B and C by selling the given articles.
- If they increase the cost price of each unit by 20%, then write the matrix representing new price.

OR

Find the total funds collected for the required purpose after 20% hike in price.

38. **Read the text carefully and answer the questions:**

[4]

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.



- Find the probability that Ajay gets Grade A in all subjects.
- Find the probability that he gets Grade A in no subjects.

**Solution**

**CBSE SAMPLE PAPER - 03**

**Class 12 - Mathematics**

**Section A**

1. (b)  $e^x \log (\sec x + \tan x) + C$

**Explanation:**  $I = \int e^x \{f(x) + f'(X)\} dx$ , where  $f(x) = \log (\sec x + \tan x)$   
 $= e^x f(x) + C = e^x \log (\sec x + \tan x) + C$

2. (a)  $2\sqrt{14}$

**Explanation:** Here, P, Q, R are collinear

$$\therefore \vec{PR} = \lambda \vec{PQ}$$

$$2\hat{i} + (y+3)\hat{j} + (z-4)\hat{k} = \lambda[6\hat{i} + 3\hat{j} + 6\hat{k}]$$

$$\Rightarrow 6\lambda = 2, y+3 = 3\lambda, z-4 = 6\lambda$$

$$\Rightarrow \lambda = \frac{1}{3}, y = -2, z = 6$$

$\therefore$  Point R(4, -2, 6)

$$\text{Now, } OR = \sqrt{(4)^2 + (-2)^2 + (6)^2} = \sqrt{56} = 2\sqrt{14}$$

3. (c)  $\vec{b} = \vec{c}$

**Explanation:**  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$  and  $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$

$$\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0 \text{ and } \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = 0$$

$$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0 \text{ and } \vec{a} \times (\vec{b} - \vec{c}) = 0$$

$$\Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$$

Also,

$$\Rightarrow |\vec{a}||\vec{b} - \vec{c}| \cos \theta = 0 \text{ and } |\vec{a}||\vec{b} - \vec{c}| \sin \theta = 0$$

$$\Rightarrow \text{If } \theta = \frac{\pi}{2} \Rightarrow \sin \theta = 1 \Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$$

4. (c)  $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

**Explanation:** If  $A \neq \phi$  and  $B \neq \emptyset$ , then by definition of conditional probability,  $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

5. (c) None of these

**Explanation: Formula :-**  $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$$\Rightarrow \int \sqrt{e^x - 1} dx$$

$$\text{Put } e^x - 1 = t \Rightarrow e^x dx = dt$$

$$\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$$

$$\text{Put } t = z^2 \quad dt = 2z dz$$

$$\Rightarrow \int \frac{2z^2}{1+z^2} dz = \int \frac{2+2z^2-2}{1+z^2} dz = 2 \int \frac{1+z^2}{1+z^2} dz - 2 \int \frac{1}{1+z^2} dz$$

$$= 2 \int dz - 2 \int \frac{1}{1+z^2} dz = 2z - 2 \tan^{-1} z + c$$

$$= 2\sqrt{t} - 2 \tan^{-1} \sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2 \tan^{-1} \sqrt{e^x - 1} + c$$

6. (b) 0.4

**Explanation:** Let A and B be independent events with  $P(A) = 0.3$  and  $P(B) = 0.4$ .  $P(B/A) = P(B) = 0.4$

7. (c) 9

**Explanation:** Required area:

$$\int_0^9 \sqrt{x} dx - \int_3^9 \left(\frac{x-3}{2}\right) dx = \left[\frac{2}{3}x^{3/2}\right]_0^9 - \frac{1}{2} \left[\frac{x^2}{2} - 3x\right]_3^9 = 9 \text{ sq.units}$$

8. (d)  $90^\circ$

**Explanation:** We have,

$$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$$

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$$

The direction ratios of the given lines are proportional to 2, 5, 4 and 1, 2, -3.

The given lines are parallel to the vectors  $\vec{b}_1 = 2\hat{i} + 5\hat{j} + 4\hat{k}$  and  $\vec{b}_2 = \hat{i} + 2\hat{j} - 3\hat{k}$

Let  $\theta$  be the angle between the given lines.

Now,

$$\begin{aligned} \cos \theta &= \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \\ &= \frac{(2\hat{i} + 5\hat{j} + 4\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{2^2 + 5^2 + 4^2} \sqrt{1^2 + 2^2 + (-3)^2}} \\ &= \frac{2 + 10 - 12}{\sqrt{45} \sqrt{14}} \\ &= 0 \\ \Rightarrow \theta &= 90^\circ \end{aligned}$$

9. (c) [0, 6]

**Explanation:** [0, 6] is the correct answer. The smallest value of  $|k\vec{a}|$  will exist at numerically smallest value of k, i.e., at  $k = 0$ , which gives  $|k\vec{a}| = |k||\vec{a}| = 0 \times 3 = 0$ . The numerically greatest value of k is 2 at which  $|k\vec{a}| = 6$ .

10. (d) Family of circles

**Explanation:** Given differential equation is

$$y \frac{dy}{dx} + x = c$$

$$\Rightarrow y dy = (c - x) dx$$

On integrating both sides, we get

$$\frac{y^2}{2} = cx - \frac{x^2}{2} + d$$

$$\Rightarrow y^2 + x^2 - 2cx - 2d = 0$$

Hence, it represents a family of circles whose centres are on the x-axis.

11. (a)  $\frac{2}{3}$

**Explanation:** Required area :

$$\begin{aligned} &= 2 \int_0^a \sqrt{4ax} dx \\ &= k\alpha (2\sqrt{4a\alpha}) \\ &= \frac{8\sqrt{a}}{3} \alpha^{\frac{3}{2}} \\ &= 4\sqrt{a} k \alpha^{\frac{3}{2}} \Rightarrow k = \frac{2}{3} \end{aligned}$$

12. (d)  $x = \tan \theta$

**Explanation:** For  $x = \tan \theta$ ,  $\tan^{-1} \left( \frac{3x - x^3}{1 - 3x^2} \right) = \tan^{-1} \left( \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) = \tan^{-1} (\tan 3\theta) = 3\theta$

13. (d) (-1, 1)

**Explanation:** We have,  $\Rightarrow f(x) = \frac{x}{x^2 + 1}$

$$\Rightarrow f'(x) = \frac{x^2 - 2x^2 + 1}{x^2 + 1}$$

$$\Rightarrow f'(x) = -\frac{x^2 - 1}{x^2 + 1}$$

$\Rightarrow$  for critical points  $f'(x) = 0$

when  $f(x) = 0$

We get  $x = 1$  or  $x = -1$

When we plot them on number line as  $f'(x)$  is multiplied by -ve sign we get

For  $x > 1$  function is decreasing

For  $x < -1$  function is decreasing

But between -1 to 1 function is increasing

$\therefore$  Function is increasing in (-1, 1)

14. (c) 512

**Explanation:**  $2^{3 \times 3} = 2^9 = 512$ .

The number of elements in a  $3 \times 3$  matrix is the product  $3 \times 3 = 9$ .

Each element can either be a 0 or a 1.

Given this, the total possible matrices that can be selected is  $2^9 = 512$

15. (d) if  $\det. A = 0$ ,  $(\text{adj } A) B = O$

**Explanation:** If  $\det. A = 0$ ,  $(\text{adj } A) B = O \Rightarrow$  The system  $AX = B$  of  $n$  equations in  $n$  unknowns may be consistent with infinitely many solutions or it may be inconsistent.

16. (b) Non-existent

**Explanation:**  $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$  is singular matrix. So its determinant value of this matrix is zero.

$$\text{i.e., } \begin{vmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{vmatrix} = 0$$

$$\Rightarrow 5(-4b + 12) - 10(-2b + 6) + 3(4 - 4) = 0$$

$$\Rightarrow -20b + 60 + 20b - 60 = 0$$

$b$  does not exist

17. (c)  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

**Explanation:**

To Find: The range of  $\sin^{-1}x$

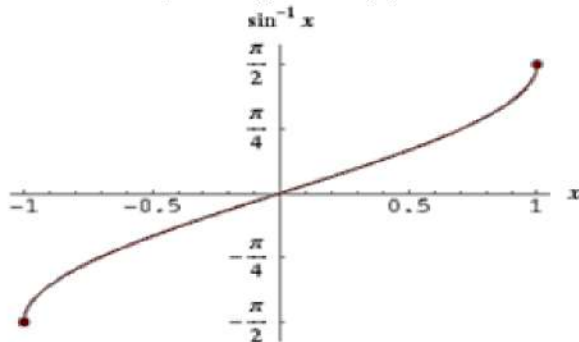
Here, the inverse function is given by  $y = f^{-1}(x)$

The graph of the function  $y = \sin^{-1}(x)$  can be obtained from the graph of

$Y = \sin x$  by interchanging  $x$  and  $y$  axes. i.e, if  $(a, b)$  is a point on  $Y = \sin x$  then  $(b, a)$  is

The point on the function  $y = \sin^{-1}(x)$

Below is the Graph of range of  $\sin^{-1}(x)$



From the graph, it is clear that the range  $\sin^{-1}(x)$  is restricted to the interval  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

18. (a)  $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$

**Explanation:** We have,  $\log\left(\frac{dy}{dx}\right) = (ax + by)$

$$\frac{dy}{dx} = e^{ax+by}$$

$$\frac{dy}{e^{by}} = e^{ax} dx$$

On integrating on both sides, we obtain

$$-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + c$$

19. (d) A is false but R is true.

**Explanation:** Let  $f(x) = 2x^3 - 24x$

$$\Rightarrow f'(x) = 6x^2 - 24 = 6(x^2 - 4)$$

$$= 6(x+2)(x-2)$$

For maxima or minima put  $f'(x) = 0$ .

$$\Rightarrow 6(x+2)(x-2) = 0$$

$$\Rightarrow x = 2, -2$$

We first consider the interval  $[1, 3]$ .

So, we have to evaluate the value of  $f$  at the critical point  $x = 2 \in [1, 3]$  and at the end points of  $[1, 3]$ .

$$\text{At } x = 1, f(1) = 2 \times 1^3 - 24 \times 1 = -22$$

$$\text{At } x = 2, f(2) = 2 \times 2^3 - 24 \times 2 = -32$$



At  $x = 3$ ,  $f(3) = 2 \times 3^3 - 24 \times 3 = -18$

$\therefore$  The absolute maximum value of  $f(x)$  in the interval  $[1, 3]$  is  $-18$  occurring at  $x = 3$ .

Hence, Assertion is false and Reason is true.

20. (d) A is false but R is true.

**Explanation: Assertion:** Minor of an element of a determinant of order  $n$  ( $n \geq 2$ ) is a determinant of order  $n - 1$ .  
So, Assertion is false.

**Reason:** We know,  $AA^{-1} = I$

$$\therefore |AA^{-1}| = |I| \Rightarrow |A| |A^{-1}| = 1$$

$$\Rightarrow |A^{-1}| = \frac{1}{|A|}$$

So, reason is true.

### Section B

21. Let  $\cot^{-1} \left( \frac{-5}{12} \right) = y$

Then  $\cot y = \frac{-5}{12}$

Now,

$$\sin \left[ 2 \cot^{-1} \left( \frac{-5}{12} \right) \right] = \sin 2y$$

$$= 2 \sin y \cos y = 2 \left( \frac{12}{13} \right) \left( \frac{-5}{13} \right) \quad [\text{since } \cot y < 0, \text{ so } y \in \left( \frac{\pi}{2}, \pi \right)]$$

$$= \frac{-120}{169}$$

22. The given equation may be written as  $\frac{dy}{dx} - \frac{1}{x}y = \frac{x+1}{x}$

This is of the form  $\frac{dy}{dx} + Py = Q$ , where  $P = -\frac{1}{x}$ ,  $Q = \frac{x+1}{x}$

$$1F = e^{\int r dx} = e^{-\int \frac{1}{x} dx} = e^{-\log|x|} = \frac{1}{x}$$

So, the required solution is given by  $y \times 1.F = \int Q \times 1.F dx$

$$\Rightarrow y \cdot e^{2x} = \int x \cdot e^{4x} \cdot e^{2x} dx$$

$$\Rightarrow y \cdot e^{2x} = \int x \cdot e^{4x} \cdot e^{2x} dx \Rightarrow y \cdot e^{2x} = \int x \cdot e^{6x} dx \Rightarrow y \cdot e^{2x} = x \int e^{6x} dx - \int \left[ -\frac{dx}{dx} \int e^{3x} dx \right] dx$$

$$\Rightarrow ye^{2x} = x \cdot \frac{e^{6x}}{6} - \int \frac{e^{6x}}{6} dx \Rightarrow ye^{2x} = \frac{x}{6} e^{6x} - \frac{e^{6x}}{36} + c$$

$$\therefore y = \frac{x e^{4x}}{6} - \frac{e^{4x}}{36} + c e^{-2x}$$

23. Given:  $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$$\Rightarrow |A| = 1$$

$$\text{Adj } A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

and,  $B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

$$|B| = -1$$

$$B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{-1} \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

$$\text{Also, } AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 14 \\ 29 & 37 \end{bmatrix}$$

$$|AB| = 407 - 406 = 1$$

$$\text{And, } \text{adj}(AB) = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\text{adj } AB}{|AB|} = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$$

$$\text{Now, } B^{-1}A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$$

$$\text{Hence, } (AB)^{-1} = B^{-1}A^{-1}$$

OR

Matrix form of given equations is  $AX = B$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Here  $|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$ ,  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  and  $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

$$= 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6)$$

$$= 10 + 15 + 15 = 40 \neq 0$$

Therefore, solution is unique and  $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A) B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Therefore,  $x = 1$ ,  $y = 2$  and  $z = -1$ .

24. Given,

$$\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k} \text{ and } \vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$$

$$\therefore \vec{a} + \vec{b} = (2\hat{i} + 2\hat{j} - 5\hat{k}) + (2\hat{i} + \hat{j} - 7\hat{k}) = 4\hat{i} + 3\hat{j} - 12\hat{k}$$

$$\Rightarrow |\vec{a} + \vec{b}| = \sqrt{4^2 + 3^2 + (-12)^2} = \sqrt{16 + 9 + 144} = \sqrt{169} = 13$$

$$\therefore \text{Required unit vector } \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{13} = \frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k}$$

25. We know, when a pair of dice is thrown, total possible outcomes are = 36

A = getting sum as 8

$$= \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$\text{Therefore, } P(A) = \frac{5}{36}$$

B = Getting number 5 atleast once =  $\{(1,5), (2,5), (3,5), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,5)\}$

$$\text{Hence } P(B) = \frac{11}{36}$$

$(A \cap B)$  = getting sum as 8 with at least one die showing 5 =  $\{(3,5), (5,3)\}$

$$\text{Hence } P(A \cap B) = \frac{2}{36}$$

Therefore the required probability is given by,  $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$$= \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$$

### Section C

26. Here, we take inverse trigonometric function as first function and algebraic function as second function.

$$\text{Let } I = \int x^2 \tan^{-1} x dx$$

$$= \int (\tan^{-1} x) \cdot x^2 dx$$

$$= (\tan^{-1} x) \cdot \frac{x^3}{3} - \int \frac{1}{1+x^2} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \frac{x^3}{x^2+1} dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \left(x - \frac{x}{x^2+1}\right) dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int x dx + \frac{1}{6} \int \frac{2x}{x^2+1} dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \log(x^2 + 1)$$

27. We have,  $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ , ( $x \neq 0$ )

This is a linear differential equation of the form  $\frac{dy}{dx} + Py = Q$ .

Here,  $P = \cot x$  and  $Q = 2x + x^2 \cot x$ .

$$\therefore \text{IF} = e^{\int P dx} = e^{\int \cot x dx} = e^{\log |\sin x|} = \sin x$$

The general solution is given by

$$\begin{aligned} y \cdot \text{IF} &= \int (\text{IF} \times Q) dx + C \\ \Rightarrow y \cdot \sin x &= \int (2x + x^2 \cot x) \sin x dx + C \\ &= 2 \int x \sin x dx + \int x^2 \cos x dx + c \\ &= 2 \int x \sin x dx + x^2 \sin x - \int 2x \sin x dx + C \\ \Rightarrow y \cdot \sin x &= x^2 \sin x + C \dots(i) \end{aligned}$$

On putting  $x = \frac{\pi}{2}$  and  $y = 0$  in Eq. (i), we get

$$0 \cdot \sin \frac{\pi}{2} = \left(\frac{\pi}{2}\right)^2 \cdot \sin \frac{\pi}{2} + C \Rightarrow C = -\frac{\pi^2}{4}$$

On putting  $C = -\frac{\pi^2}{4}$  in Eq. (i), we get

$$y \cdot \sin x = x^2 \sin x - \frac{\pi^2}{4}$$

$$\therefore y = x^2 - \frac{\pi^2}{4} \operatorname{cosec} x$$

[dividing both sides by  $\sin x$ ]

OR

We have,

$$x(1+y^2) dx - y(1+x^2) dy = 0$$

$$\Rightarrow x(1+y^2) dx = y(1+x^2) dy$$

$$\Rightarrow \frac{x}{1+x^2} dx = \frac{y}{1+y^2} dy$$

$$\Rightarrow \frac{2x}{1+x^2} dx = \frac{2y}{1+y^2} dy$$

Integrating both sides, we get

$$\int \frac{2x}{1+x^2} dx = \int \frac{2y}{1+y^2} dy$$

$$\Rightarrow \log |1+x^2| = \log |1+y^2| + \log C$$

$$\Rightarrow \log \left| \frac{1+x^2}{1+y^2} \right| = \log C$$

$$\Rightarrow \frac{1+x^2}{1+y^2} = C$$

$$\Rightarrow (1+x^2) = (1+y^2) C \dots(i)$$

It is given that when  $x = 1$ ,  $y = 0$ . So, putting  $x = 1$  and  $y = 0$  in (i), we get

$$(1+1) = (1+0) C \Rightarrow C = 2$$

Putting  $C = 2$  in (i), we get

$$(1+x^2) = 2(1+y^2),$$

Which is the required solution.

28. According to the question,

$$\text{Given, } \vec{OA} = 4\hat{i} + 5\hat{j} + \hat{k},$$

$$\vec{OB} = -\hat{j} - \hat{k},$$

$$\vec{OC} = 3\hat{i} + \lambda\hat{j} + 4\hat{k} \text{ and}$$

$$\vec{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}.$$

$$\text{Now, } \vec{AB} = \vec{OB} - \vec{OA} = -\hat{j} - \hat{k} - (4\hat{i} + 5\hat{j} + \hat{k})$$

$$= -4\hat{i} - 6\hat{j} - 2\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = 3\hat{i} + \lambda\hat{j} + 4\hat{k} - (4\hat{i} + 5\hat{j} + \hat{k})$$

$$= -\hat{i} + (\lambda - 5)\hat{j} + 3\hat{k}$$

$$\text{and } \vec{AD} = \vec{OD} - \vec{OA}$$

$$= -4\hat{i} + 4\hat{j} + 4\hat{k} - (4\hat{i} + 5\hat{j} + \hat{k})$$

$$= -8\hat{i} - \hat{j} + 3\hat{k}$$

Since, vectors  $\vec{OA}$ ,  $\vec{OB}$ ,  $\vec{OC}$  and  $\vec{OD}$  are coplanar.

$$\therefore [\vec{AB} \vec{AC} \vec{AD}] = 0$$

$$\therefore \begin{vmatrix} -4 & -6 & -2 \\ -1 & (\lambda - 5) & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$

$$-4(3\lambda - 15 + 3) + 6(-3 + 24) - 2(1 + 8\lambda - 40) = 0$$

$$\Rightarrow -4(3\lambda - 12) + 6(21) - 2(8\lambda - 39) = 0$$

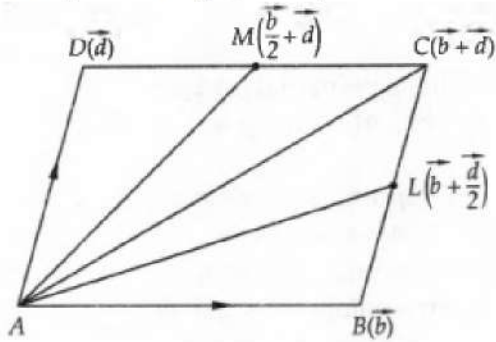
$$\Rightarrow -12\lambda + 48 + 126 - 16\lambda + 78 = 0$$

$$\Rightarrow -28\lambda + 252 = 0$$

$$\Rightarrow \lambda = 9$$

OR

Taking A as the origin, let the position vectors of vertices B and D of parallelogram ABCD be  $\vec{b}$  and  $\vec{d}$  respectively.



From figure,

$$\vec{AB} = \vec{b} \text{ and } \vec{AD} = \vec{d}.$$

In triangle ABC, we have

$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\Rightarrow \vec{AC} = \vec{AB} + \vec{AD} \quad [\because \vec{BC} = \vec{AD}]$$

$$\Rightarrow \vec{AC} = \vec{b} + \vec{d}$$

$\therefore$  Position vector of C is  $\vec{b} + \vec{d}$

Since L and M are mid-points of BC and CD respectively. Therefore,

$$\text{Position vector of } L = \frac{\vec{b} + (\vec{b} + \vec{d})}{2} = \vec{b} + \frac{1}{2}\vec{d}, \text{ Position vector of } M = \frac{(\vec{b} + \vec{d}) + \vec{d}}{2} = \frac{\vec{b}}{2} + \vec{d}$$

$$\therefore \vec{AL} = \text{Position vector of } L - \text{Position vector of } A = \vec{b} + \frac{1}{2}\vec{d} - \vec{0} = \vec{b} + \frac{1}{2}\vec{d} = \vec{AB} + \frac{1}{2}\vec{AD}$$

$$\text{and } \vec{AM} = \text{Position vector of } M - \text{Position vector of } A = \frac{\vec{b}}{2} + \vec{d} - \vec{0} = \frac{\vec{b}}{2} + \vec{d} = \frac{1}{2}\vec{AB} + \vec{AD}$$

$$\therefore \vec{AL} + \vec{AM} = \left(\vec{b} + \frac{1}{2}\vec{d}\right) + \left(\frac{1}{2}\vec{b} + \vec{d}\right) = \frac{3}{2}\vec{b} + \frac{3}{2}\vec{d} = \frac{3}{2}(\vec{b} + \vec{d}) = \frac{3}{2}\vec{AC}$$

29. Let the given integral be,  $y = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx$

Using theorem of definite integral

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$y = \int_0^{\pi/2} \frac{\sin^2 \left(\frac{\pi}{2} - x\right)}{\sin^2 \left(\frac{\pi}{2} - x\right) + \cos^2 \left(\frac{\pi}{2} - x\right)} dx$$

$$y = \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx \dots \text{(ii)}$$

Adding equation (i) and equation (ii) we get.

$$2y = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin^2 x + \cos^2 x} dx$$

$$2y = \int_0^{\pi/2} 1 dx$$

$$2y = (x)_0^{\pi/2}$$

$$y = \frac{\pi}{4}$$

OR

Let the given integral be,

$$I = \int \frac{dx}{x(x^4-1)} dx$$

Putting  $t = x^4$

$$dt = 4x^3 dx$$

$$I = \int \frac{x^3 dx}{x^4(x^4-1)} = \frac{1}{4} \times \int \frac{dt}{t(t-1)}$$

Now putting,  $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$  .... (1)

$$A(t-1) + Bt = 1$$

Now put  $t-1 = 0$

Therefore,  $t = 1$

$$A(0) + B(1) = 1$$

$$B = 1$$

Now put  $t = 0$

$$A(0-1) + B(0) = 1$$

$$A = -1$$

Now From equation (1) we get,

$$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\therefore \frac{1}{4} \int \frac{1}{t(t-1)} dt = -\frac{1}{4} \int \frac{1}{t} dt + \frac{1}{4} \int \frac{1}{t-1} dt$$

$$= -\frac{1}{4} \log t + \frac{1}{4} \log |t-1| + c$$

$$= -\frac{1}{4} \log x^4 + \frac{1}{4} \log |x^4 - 1| + c$$

$$= -\log |x| + \frac{1}{4} \log |x^4 - 1| + c$$

30.  $x = ae^t (\sin t + \cos t)$  and  $y = ae^t (\sin t - \cos t)$  (Given)

$$\therefore \frac{dx}{dt} = a[e^t(\cos t - \sin t) + e^t(\sin t + \cos t)]$$

$$= -y + x$$

$$\text{and } \frac{dy}{dt} = a[e^t(\sin t - \cos t) + e^t(\sin t + \cos t)]$$

$$= y + x$$

Therefore

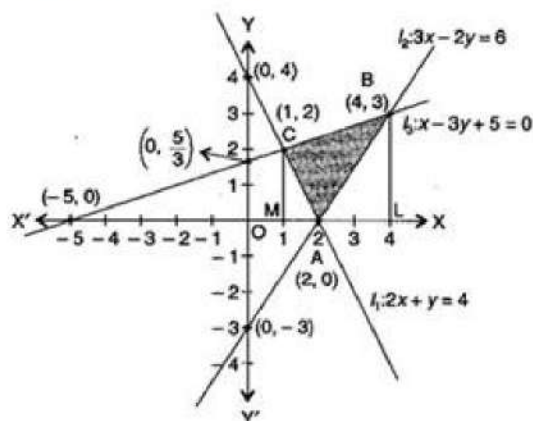
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\text{OR } \frac{dy}{dx} = \frac{x+y}{x-y}$$

31. Equation of one line  $l_1$  is  $2x + y = 4$ ,

Equation of second line  $l_2$  is  $3x - 2y = 6$

And Equation of third line  $l_3$  is  $x - 3y + 5 = 0$



Here, vertices of triangle ABC are A (2, 0), B (4, 3) and C (1, 2).

Now, Required area of triangle

$$= \text{Area of trapezium CLMB} - \text{Area } \triangle ACM - \text{Area } \triangle ABL$$

$$= \left| \int_1^4 \frac{1}{3}(x+5) dx \right| - \left| \int_1^2 (4-2x) dx \right| - \left| \int_2^4 \frac{3}{2}(x-2) dx \right|$$

$$= \frac{1}{3} \left| \left( \frac{x^2}{2} + 5x \right)_1^4 \right| - \left| \left( 4x - \frac{2x^2}{2} \right)_1^2 \right| - \frac{3}{2} \left| \left( \frac{x^2}{2} - 2x \right)_2^4 \right|$$

$$= \frac{1}{3} \left[ 8 + 20 - \left( \frac{1}{2} + 5 \right) \right] - \{ (8-4) - (4-1) \} - \frac{3}{2} \{ (8-8) - (2-4) \}$$

$$\begin{aligned}
 &= \frac{1}{3} \left( 28 - \frac{11}{2} \right) - (4 - 3) - \frac{3}{2} \times 2 \\
 &= \frac{1}{3} \times \frac{45}{2} - 1 - 3 \\
 &= \frac{15}{2} - 1 - 3 = \frac{7}{2} \text{ sq. units}
 \end{aligned}$$

### Section D

32. We have

$$x + 2y \geq 10 \dots (i)$$

$$x + y \geq 6 \dots (ii)$$

$$3x + y \geq 8 \dots (iii)$$

$$x, y \geq 0$$

First of all find point of intersection by taking  $x + 2y = 10$

$$x + y = 6$$

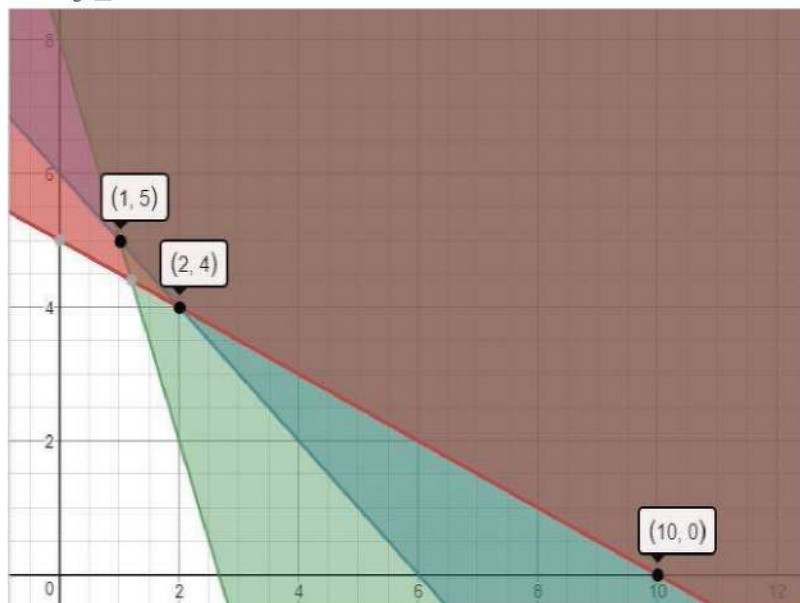
$$3x + y = 8$$

On solving these equation simultaneously you will get point of intersection.

$$x + 2y \geq 10$$

$$x + y \geq 6$$

$$3x + y \geq 8$$



$$Z = 3x + 5y$$

Put (1, 5)

$$Z = 3 + 25 = 28$$

Put (2, 4)

$$Z = 6 + 20 = 26$$

Hence minimum value of  $Z$  is 26.

33. Given that  $A = [-1, 1]$

i.  $f(x) = \frac{x}{2}$

Let  $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$$

So,  $f(x)$  is one-one.

Now, let  $y = \frac{x}{2}$

$$\Rightarrow x = 2y \notin A, \forall y \in A$$

As for  $y = 1 \in A, x = 2 \notin A$

So,  $f(x)$  is not onto.

Also,  $f(x)$  is not bijective as it is not onto.

ii.  $g(x) = |x|$

Let  $g(x_1) = g(x_2)$

$$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$$

So,  $g(x)$  is not one-one.

Now,  $x = \pm y \notin A$  for all  $y \in \mathbb{R}$

So,  $g(x)$  is not onto, also,  $g(x)$  is not bijective.

iii.  $h(x) = x|x|$

$$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$$

So,  $h(x)$  is one-one

Now, let  $y = x|x|$

$$\Rightarrow y = x^2 \in A, \forall x \in A$$

So,  $h(x)$  is onto also,  $h(x)$  is a bijective.

iv.  $k(x) = x^2$

Let  $k(x_1) = k(x_2)$

$$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$$

Thus,  $k(x)$  is not one-one.

Now, let  $y = x^2$

$$\Rightarrow x\sqrt{y} \notin A, \forall y \in A \quad x = \sqrt{y} \notin A, \forall y \in A$$

As for  $y = -1$ ,  $x = \sqrt{-1} \notin A$

Hence,  $k(x)$  is neither one-one nor onto.

OR

Here  $R$  is a relation on  $N \times N$ , defined by  $(a, b) R (c, d) \Leftrightarrow a + d = b + c$  for all  $(a, b), (c, d) \in N \times N$

We shall show that  $R$  satisfies the following properties

i. Reflexivity:

We know that  $a + b = b + a$  for all  $a, b \in N$ .

$$\therefore (a, b) R (a, b) \text{ for all } (a, b) \in (N \times N)$$

So,  $R$  is reflexive.

ii. Symmetry:

Let  $(a, b) R (c, d)$ . Then,

$$(a, b) R (c, d) \Rightarrow a + d = b + c$$

$$\Rightarrow c + b = d + a$$

$$\Rightarrow (c, d) R (a, b).$$

$$\therefore (a, b) R (c, d) \Rightarrow (c, d) R (a, b) \text{ for all } (a, b), (c, d) \in N \times N$$

This shows that  $R$  is symmetric.

iii. Transitivity:

Let  $(a, b) R (c, d)$  and  $(c, d) R (e, f)$ . Then,

$$(a, b) R (c, d) \text{ and } (c, d) R (e, f)$$

$$\Rightarrow a + d = b + c \text{ and } c + f = d + e$$

$$\Rightarrow a + d + c + f = b + c + d + e$$

$$\Rightarrow a + f = b + e$$

$$\Rightarrow (a, b) R (e, f).$$

Thus,  $(a, b) R (c, d)$  and  $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$

This shows that  $R$  is transitive.

$\therefore R$  is reflexive, symmetric and transitive

Hence,  $R$  is an equivalence relation on  $N \times N$

34. Suppose,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

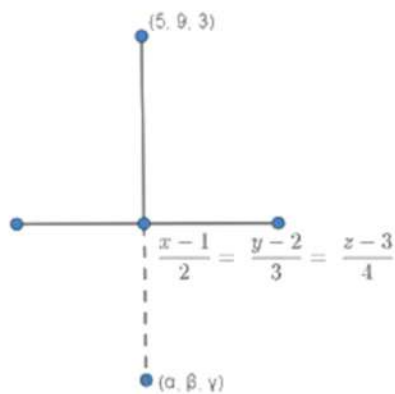
So the foot of the perpendicular is  $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

The direction ratios of the perpendicular is

$$(2\lambda + 1 - 5) : (3\lambda + 2 - 9) : (4\lambda + 3 - 3)$$

$$\Rightarrow (2\lambda - 4) : (3\lambda - 7) : (4\lambda)$$

Direction ratio of the line is  $2 : 3 : 4$



From the direction ratio of the line and the direct ratio of its perpendicular, we have

$$2(2\lambda - 4) + 3(3\lambda - 7) + 4(4\lambda) = 0$$

$$\Rightarrow 4\lambda - 8 + 9\lambda - 21 + 16\lambda = 0$$

$$\Rightarrow 29\lambda = 29$$

$$\Rightarrow \lambda = 1$$

Therefore, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and  $(\alpha, \beta, \gamma)$

Therefore, we have

$$\frac{\alpha+5}{2} = 3 \Rightarrow \alpha = 1$$

$$\frac{\beta+9}{2} = 5 \Rightarrow \beta = 1$$

$$\frac{\gamma+3}{2} = 7 \Rightarrow \gamma = 11$$

Therefore, the image is (1, 1, 11)

OR

The vector equation of the line through point A and B is

$$\vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda [(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k}]$$

$$\Rightarrow \vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda (2\hat{i} - 3\hat{j} + 5\hat{k})$$

$$\vec{r} = (3+2\lambda)\hat{i} + (4-3\lambda)\hat{j} + (1+5\lambda)\hat{k} \dots(1)$$

Let P be the point where the line AB crosses the XY plane. Then the position vector  $\vec{r}$  of the point P is the form  $x\hat{i} + y\hat{j}$

Then, from (1), we have,

$$x\hat{i} + y\hat{j} = (3+2\lambda)\hat{i} + (4-3\lambda)\hat{j} + (1+5\lambda)\hat{k}$$

$$\Rightarrow x = 3+2\lambda, y = 4-3\lambda, 1+5\lambda = 0$$

$$\text{Now, } 1+5\lambda \text{ gives, } \lambda = -\frac{1}{5}$$

$$\therefore x = 3 + 2\left(-\frac{1}{5}\right) \text{ and } y = 4 - 3\left(-\frac{1}{5}\right)$$

$$\Rightarrow x = \frac{13}{5} \text{ and } y = \frac{23}{5}$$

$$\text{Hence the required point is } \left(\frac{13}{5}, \frac{23}{5}, 0\right)$$

$$35. \text{LHL} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{h \rightarrow 0} f(0-h)$$

$$= \lim_{h \rightarrow 0} (-h)^m \sin\left(-\frac{1}{h}\right)$$

$$= \lim_{h \rightarrow 0} -(-h)^m \sin\left(\frac{1}{h}\right)$$

$$= 0 \times k \text{ [when } -1 \leq k \leq 1 \text{]}$$

$$= 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{h \rightarrow 0} f(0+h)$$

$$= \lim_{h \rightarrow 0} (0+h)^m \sin\left(\frac{1}{0+h}\right)$$

$$= \lim_{h \rightarrow 0} (h)^m \sin\left(\frac{1}{h}\right)$$

$$= 0 \times k \text{ [when } -1 \leq k' \leq 1 \text{]}$$

$$= 0$$



$$\text{LHL} = \text{RHL} = f(0)$$

Since,  $f(x)$  is continuous at  $x = 0$

For Differentiability at  $x = 0$

$$(\text{LHD at } x = 0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{h \rightarrow 0^-} \frac{f(0-h) - f(0)}{(0-h) - 0}$$

$$= \lim_{h \rightarrow 0^-} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{h}$$

$$= \lim_{h \rightarrow 0^-} -(-h)^{m-1} \sin\left(\frac{1}{h}\right)$$

= Not Defined [since  $0 < m < 1$ ]

$$(\text{RHD at } x = 0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{(0+h) - 0}$$

$$= \lim_{h \rightarrow 0^+} \frac{h^m \sin\left(\frac{1}{h}\right)}{0+h-0}$$

$$= \lim_{h \rightarrow 0^+} (h)^{m-1} \sin\left(\frac{1}{h}\right)$$

$$= 0$$

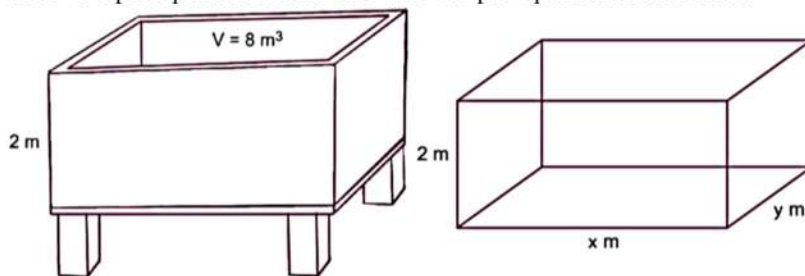
Since,  $(\text{LHD at } x = 0) \neq (\text{RHD at } x = 0)$

**Hence,  $f(x)$  is continuous at  $x = 0$  but not differentiable.**

### Section E

36. Read the text carefully and answer the questions:

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is  $8 \text{ m}^3$  as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.



(i) Since 'C' is cost of making tank

$$\therefore C = 70xy + 45 \times 2(2x + 2y)$$

$$\Rightarrow C = 70xy + 90(2x + 2y)$$

$$\Rightarrow C = 70xy + 180(x + y) \quad [\because 2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}]$$

$$\Rightarrow C = 70x \times \frac{4}{x} + 180 \left(x + \frac{4}{x}\right)$$

$$\Rightarrow C = 280 + 180 \left(x + \frac{4}{x}\right)$$

(ii)  $x \cdot y = 4$

Volume of tank = length  $\times$  breadth  $\times$  height (Depth)

$$8 = x \cdot y \cdot 2$$

$$\Rightarrow 2xy = 8 \Rightarrow xy = 4$$

(iii) For maximum or minimum

$$\frac{dC}{dx} = 0$$

$$\frac{d}{dx} \left(280 + 180 \left(x + \frac{4}{x}\right)\right) = 0 \Rightarrow 180 \left(1 + 4 \left(-\frac{1}{x^2}\right)\right) = 0$$

$$\Rightarrow 180 \left(1 - \frac{4}{x^2}\right) = 0 \Rightarrow 1 - \frac{4}{x^2} = 0$$

$$\Rightarrow \frac{4}{x^2} = 1 \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$$\Rightarrow x = 2 \text{ (length can never be negative)}$$

OR

$$\text{Now, } \frac{d^2C}{dx^2} = 180 \left( + \frac{8}{x^3} \right)$$

$$\Rightarrow \frac{d^2C}{dx^2} \Big|_{x=2} = 180 \times \frac{8}{8} = 180 = +ve$$

Hence, to minimize C,  $x = 2m$

37. Read the text carefully and answer the questions:

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.



Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

(i)

$$P = \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} \text{Fans} & \text{Mats} & \text{Plates} \\ 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}$$

$$Q = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

(ii) Clearly, total funds collected by each school is given by the matrix

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

∴ Funds collected by school A is ₹7000.

Funds collected by school B is ₹6125.

Funds collected by school C is ₹7875.

(iii)

$$\text{New price matrix } Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$\text{New price matrix } Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

OR

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix}$$

$$PQ = \begin{bmatrix} 1200 + 6000 + 1200 \\ 750 + 4800 + 1800 \\ 1050 + 6000 + 2400 \end{bmatrix} = \begin{bmatrix} 8400 \\ 7350 \\ 9450 \end{bmatrix}$$

Total fund collected =  $8400 + 7350 + 9450 = ₹25,200$

**38. Read the text carefully and answer the questions:**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.



- (i)  $P(\text{Grade A in Maths}) = P(M) = 0.2$   
 $P(\text{Grade A in Physics}) = P(P) = 0.3$   
 $P(\text{Grade A in Chemistry}) = P(C) = 0.5$   
 $P(\text{Grade A in all subjects}) = P(M \cap P \cap C) = P(M) \cdot P(P) \cdot P(C)$   
 $P(\text{Grade A in all subjects}) = 0.2 \times 0.3 \times 0.5 = 0.03$
- (ii)  $P(\text{Grade A in Maths}) = P(M) = 0.2$   
 $P(\text{Grade A in Physics}) = P(P) = 0.3$   
 $P(\text{Grade A in Chemistry}) = P(C) = 0.5$   
 $P(\text{Grade A in no subjects}) = P(\bar{M} \cap \bar{P} \cap \bar{C}) = P(\bar{M}) \cdot P(\bar{P}) \cdot P(\bar{C})$   
 $P(\text{Grade A in no subjects}) = 0.8 \times 0.7 \times 0.5 = 0.280$