CBSE SAMPLE PAPER - 10

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If (a, b, c) is the image of the point (1, 2, -3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a + b + c is equals to: **[1]**

   a) -1 b) 1

   c) 2 d) 3

2. $\int \frac{x^2}{(9+4x^2)} = ?$ **[1]**

   a) None of these b) $\frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{2x}{3} + C$

   c) $\frac{x}{4} - \frac{1}{8}\tan^{-1}\frac{x}{3} + C$ d) $\frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{x}{3} + C$

3. $(\vec{a}+\vec{b}) \cdot (\vec{b}+\vec{c}) \times (\vec{a}+\vec{b}+\vec{c}) =$ **[1]**

   a) 0 b) $-[\vec{a}\vec{b}\vec{c}]$

   c) $2[\vec{a}\vec{b}\vec{c}]$ d) $\begin{bmatrix}\vec{a} & \vec{b} & \vec{c}\end{bmatrix}$

4. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3 \text{ and } x^2 \leq 4y\}$ is: **[1]**

   a) $\frac{5}{3}$ b) $\frac{10}{3}$

   c) $\frac{8}{3}$ d) $\frac{7}{3}$

5. The area (in sq units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is **[1]**

   a) $\frac{3}{2} - \frac{1}{\log_e 2}$ b) $\frac{1}{2}$

   c) $\frac{3}{2}$ d) $\log_e 2 + \frac{3}{2}$

6. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is **[1]**

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{3}{4}$

d) $\frac{1}{8}$

7. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? **[1]**

a) $\frac{1}{52}$

b) $\frac{3}{52}$

c) $\frac{5}{52}$

d) $\frac{1}{4}$

8. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is Z = 4x + 3y. **[1]**

Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325
(A) The quantity in column A is greater	
(B) The quantity in column B is greater	
(C) The two quantities are equal	
(D) The relationship can not be determined on the basis of the information supplied	

a) The quantity in column A is greater

b) The quantity in column B is greater

c) The two quantities are equal

d) The relationship can not be determined on the basis of the information supplied

9. What is the area of △OAB, where O is the origin, $\overrightarrow{OA} = 3\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} + \hat{j} - 3\hat{k}$? **[1]**

a) $\sqrt{30}$ sq units

b) $\frac{5\sqrt{6}}{2}$ sq units

c) $5\sqrt{6}$ sq units

d) $\sqrt{6}$ sq units

10. Find a particular solution of $x\left(x^2 - 1\right)\frac{dx}{dy} = 1$; y = 0 when x = 2. **[1]**

a) $y = \frac{1}{2}\log\left(\frac{x^3-1}{x^2}\right) - \frac{1}{2}log\frac{3}{4}$

b) $y = \frac{1}{2}\log\left(\frac{x^2-1}{x^3}\right) + \frac{1}{2}log\frac{3}{4}$

c) $y = \frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right) - \frac{1}{2}log\frac{3}{7}$

d) $\frac{x^4}{4} - \frac{x^2}{2} = y + 2$

11. The solution of the differential equation = x dx + y dy = x^2y dy - y^2x dx is **[1]**

a) $x^3 + 1 = C(1 - y^3)$

b) $x^3 - 1 = C(1 + y^3)$

c) $x^2 + 1 = C(1 - y^2)$

d) $x^2 - 1 = C\left(1 + y^2\right)$

12. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$ is: **[1]**

a) π

b) $\frac{\pi}{4}$

c) $\frac{\pi}{2}$

d) $\frac{3\pi}{2}$

13. The solution of $x^2 + y^2\frac{dy}{dx} = 4$, is **[1]**

a)

b)

$x^3 + y^3 = 12x + C$ $x^3 + y^3 = 3x + C$

c) $x^2 + y^2 = 12x + C$ d) $x^2 + y^2 = 3x + C$

14. The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ \frac{k}{2}, & x = 0 \end{cases}$ is continuous at x = 0, then k = **[1]**

a) 3 b) 12

c) 9 d) 6

15. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (adj(adj A)) is **[1]**

a) 14^3 b) 14

c) 14^4 d) 14^2

16. Let y = y(x) be a solution of the differential equation, $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0$, |x| < 1. If y $\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then y $\left(\frac{-1}{\sqrt{2}}\right)$ is equal to: **[1]**

a) $\frac{1}{\sqrt{2}}$ b) $\frac{\sqrt{3}}{2}$

c) $-\frac{\sqrt{3}}{2}$ d) $-\frac{1}{\sqrt{2}}$

17. The value of the expression sin [$\cot^{-1}$ (cos ($\tan^{-1}$ 1))] is **[1]**

a) $\sqrt{\frac{2}{3}}$ b) 0

c) $\frac{1}{\sqrt{3}}$ d) 1

18. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is **[1]**

a) perpendicular to z-axis b) parallel to z-axis

c) parallel to y-axis d) parallel to x-axis

19. **Assertion (A):** The maximum value of Z = x + 3y. Such that 2x + y $\leq$ 20, x + 2y $\leq$ 20, x, y $\geq$ 0 is 30. **[1]**

Reason (R): The variables that enter into the problem are called decision variables.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. f(x) = [x - 1] + |x - 2|, where [·] denotes the greatest integer function. **[1]**

Assertion (A): f(x) is discontinuous at x = 2.

Reason (R): f(x) is non derivable at x = 2.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Solve the differential equations: $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y + y\cos y}$ **[2]**

22. Examine the continuity of f, where f is defined by $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$ **[2]**

23. If a line makes angles $90^0, 135^0, 45^0$ with the x, y and z - axes respectively, find its direction cosines. **[2]**

OR

The Cartesian equations of a line are $\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$. Find the vector equation of the line.

24. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin? **[2]**

25. For the principal value, evaluate $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$. **[2]**

Section C

26. Maximize Z = x + 2y subject to the constraints **[3]**

$x - y \geq 0, 2y \leq x + 2, x, y \geq 0$

27. Using integration, find the area of the region bounded by the line y - 1 = x, the x -axis and the ordinates x = - 2 and x = 3. **[3]**

OR

If S be the area of the region enclosed by $y = e^{-x^2}$, y = 0 and x = 1, then show that

$1 - \frac{1}{e} \leq S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

28. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$ **[3]**

OR

Evaluate : $\int_0^1 \frac{1}{1+2x+2x^2+2x^3+x^4} dx$

29. Prove that the lines $x = py + q, z = ry + s$ and $x = p'y + q, z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$. **[3]**

OR

Find the vector equation of a line passing through a point with position vector $2\hat{i} - \hat{j} + \hat{k}$, and parallel to the lines joining $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$. Also, find the cartesian equivalent of this equation.

30. Sketch the graph y = $|x-5|$. Evaluate $\int_0^1 |x-5|dx$. What does this value of the integral represent on the graph. **[3]**

31. If $y = a\{x + \sqrt{x^2+1}\}^n + b\{x - \sqrt{x^2+1}\}^{-n}$, prove that $\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0$ **[3]**

Section D

32. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$. **[5]**

33. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that A (adj A) = (adj A) A= |A| I_3. **[5]**

OR

By using determinants, solve the following system of equations:

x + y + z = 1

x + 2y + 3z = 4

x + 3y + 5z = 7

34. Given, A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following: **[5]**

a. an injective mapping from A to B

b. a mapping from A to B which is not injective

c. a mapping from B to A.

OR

Let A = R - {3}, B = R - {1]. If $f : A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$ $\forall x \in A$. Then, show that f is bijective.

35. Evaluate the definite integral $\int_0^{\pi} \frac{x \tan x}{\sec x+\tan x} dx$. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

The Government declare that farmers can get ₹300 per quintal for their onions on 1st July and after that, the price will be dropped by ₹3 per quintal per extra day. Govind's father has 80 quintals of onions in the field on 1st July and he estimates that the crop is increasing at the rate of 1 quintal per day.

(i) If x is the number of days after 1st July, then express price and quantity of onion and the revenue as a function of x.

(ii) Find the number of days after 1st July, when Govind's father attains maximum revenue.

(iii) On which day should Govind's father harvest the onions to maximize his revenue?

OR

Find the maximum revenue collected by Govind's father.

37. **Read the text carefully and answer the questions:** **[4]**

On her birthday, Seema decided to donate some money to the children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹).

(i) Represent given information in matrix algebra.

(ii) Find the adjoint of Matrix containing information about of number of children and amount she paid?

(iii) Find the number of children who were given some money by Seema?

OR

How much amount does Seema spend in distributing the money to all the students of the Orphanage?

38. **Read the text carefully and answer the questions:** **[4]**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) Find the probability that she gets grade A in all subjects.

(ii) Find the probability that she gets grade A in no subjects.

Solution

CBSE SAMPLE PAPER - 10

Class 12 - Mathematics

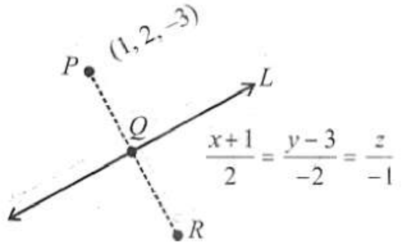
Section A

1. **(c)** 2

Explanation:

$\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}=\lambda$

Any point on line = Q(2λ -1, -2λ + 3, -λ)



$\therefore$ D.r. of PQ = [2λ - 2, - 2λ + 1, -λ + 3]

D.r. of given line = [2, -2, -1]

$\because$ PQ is perpendicular to line L

$\therefore$ 2(2λ - 2) - 2(-2λ + 1) - 1(-λ + 3) = 0

$\Rightarrow$ 4λ -4 + 4λ - 2 + λ - 3 = 0

$\Rightarrow$ 9λ - 9 = 0 $\Rightarrow$ λ = 1

$\because$ Q is mid point of PR = Q = (1, 1, -1)

$\therefore$ Coordinate of image R = (1, 0, 1) = (a, b, c)

$\therefore$ a + b + c = 2

2. **(b)** $\frac{x}{4}-\frac{3}{8}\tan^{-1}\frac{2x}{3}+C$

Explanation: $I=\int\frac{x^2}{4x^2+9}$

$=\frac{1}{4}\int\frac{4x^2+9-9}{4x^2+9}dx$

$=\frac{1}{4}\int 1+\frac{1}{4}\int\frac{-9}{4x^2+9}dx$

$=\frac{x}{4}-\frac{9}{4}\int\frac{1}{(2x)^2+3^2}dx$

Let 2x = t $\Rightarrow$ 2 dx = dt

$\therefore I=\frac{x}{4}-\frac{9}{8}\int\frac{1}{(t)^2+3^2}dx$

We know, $\int\frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$\Rightarrow I=\frac{x}{4}-\frac{9}{4.2.3}\tan^{-1}\frac{t}{3}+c$

put t = 2x

$=\frac{x}{4}-\frac{3}{8}\tan^{-1}\frac{2x}{3}+c.$

Which is the required solution.

3. **(d)** $\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$

Explanation: $(\vec{a}+\vec{b})\cdot(\vec{b}+\vec{c})\times(\vec{a}+\vec{b}+\vec{c})$

=$(\vec{a}+\vec{b}).\,(\vec{b}\times(\vec{a}+\vec{b}+\vec{c})+\vec{c}\times(\vec{a}+\vec{b}+\vec{c}))$

$=(\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{a}+\vec{b}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}+\vec{c}\times\vec{c})$

$=(\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{a}+0+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}+0)$

$=(\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{a}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}-\vec{b}\times\vec{c})$

$=(\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{a}+\vec{c}\times\vec{a})$

$=\vec{a}\cdot(\vec{b}\times\vec{a})+\vec{a}\cdot(\vec{c}\times\vec{a})+\vec{b}\cdot(\vec{b}\times\vec{a})+\vec{b}\cdot(\vec{c}\times\vec{a})$

$=0+0+0+[\vec{b}\vec{c}\vec{a}]$

= $[\vec{a}\vec{b}\vec{c}]$

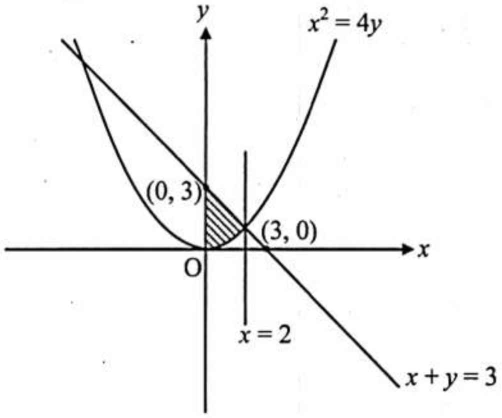
4. **(b)** $\frac{10}{3}$

Explanation: $x^2 = 4y$...(i)

$x + y = 3$ ...(ii)

Solving (i) and (ii), we get

$x = -6, x = 2$



Required area = $\int_0^2 \left[(3-x)-\frac{x^2}{4}\right]dx = \left[3x-\frac{x^2}{2}-\frac{x^3}{12}\right]_0^2$

$= \left(6-2-\frac{8}{12}\right) - (0)$

$= \frac{40}{12}$

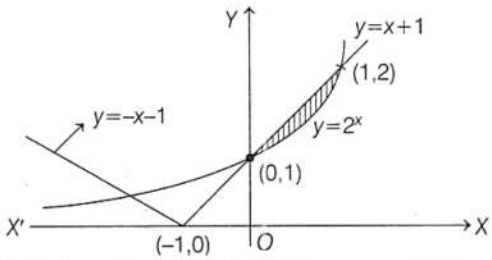
$= \frac{10}{3}$ sq. units

5. **(a)** $\frac{3}{2}-\frac{1}{\log_e 2}$

Explanation: Given, equations of curves

$y = 2^x$ and $y=|x+1|=\begin{cases} x+1 & ,x\geq -1 \\ -x-1 & ,x<-1 \end{cases}$

$\because$ The figure of above given curves is



In first quadrant, the above given curves intersect each other at (1, 2).

So, the required area = $\int_0^1 \left((x+1)-2^x\right)dx$

$= \left[\frac{x^2}{2}+x-\frac{2^x}{\log_e 2}\right]_0^1 \left[\because \int a^x dx = \frac{a^x}{\log_e a}+C\right]$

$= \left[\frac{1}{2}+1-\frac{2}{\log_e 2}+\frac{1}{\log_e 2}\right]$

$= \frac{3}{2}-\frac{1}{\log_e 2}$

6. **(d)** $\frac{1}{8}$

Explanation: Let E_1 = Event for getting an even number on the die

And E_2 = Event that a spade card is selected

$\therefore P(E_1)=\frac{3}{6}=\frac{1}{2}$ and $P(E_2)=\frac{13}{52}=\frac{1}{4}$

Then, $P(E_1\cap E_2)=P(E_1)\cdot P(E_2)=\frac{1}{2}\cdot\frac{1}{4}=\frac{1}{8}$

7. **(a)** $\frac{1}{52}$

Explanation: Let, E_1, E_2 and E_3 are events of selection of a scooter driver, car driver and truck driver respectively.

$\therefore P(E_1)=\frac{2000}{12000}=\frac{1}{6}$, $P(E_2)=\frac{4000}{12000}=\frac{1}{3}$,$P(E_3)=\frac{6000}{12000}=\frac{1}{2}$.

Let A = event that the insured person meet with the accident.

$\therefore P(A|E_1) = 0.01, P(A|E_2) = 0.03, P(A|E_3) = 0.15$

$\Rightarrow$ P(E$_1$|A) = $\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$

$= \frac{\frac{1}{6}\times0.01}{\frac{1}{6}\times0.01+\frac{1}{3}\times0.03+\frac{1}{2}\times0.15} = \frac{1}{52}$

8. **(b)** The quantity in column B is greater

Explanation:

Corner Points	Corresponding Value of Z = 4x + 3y
(0, 0)	0
(0, 40)	120
(20, 40)	200
(60, 20)	300 **(Maximum)**
(60, 0)	240

So, maximum value of Z = 300 < 325.

Therefore, the quantity in column B is greater.

Which is the required solution.

9. **(b)** $\frac{5\sqrt{6}}{2}$ sq units

Explanation: Since, area of $\triangle$OAB = $\frac{1}{2}|OA \times OB|$

$\therefore \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix}$

$= 2\hat{i} + 11\hat{j} + 5\hat{k}$

$\therefore |\text{OA} \times \text{OB}| = \sqrt{2^2 + 11^2 + 5^2} = 5\sqrt{6}$

$\therefore$ Required area = $\frac{1}{2} \times 5\sqrt{6} = \frac{5\sqrt{6}}{2}$ sq units

10. **(d)** $\frac{x^4}{4} - \frac{x^2}{2} = y + 2$

Explanation: $x(x^2-1)dx = dy$

$\int(x^3 - x)dx = \int dy$

$\frac{x^4}{4} - \frac{x^2}{2} = y + c$

Here y = 0 when x = 2

$\frac{2^4}{4} - \frac{2^2}{2} = 0 + c$

$4 - 2 = c$

$\therefore c = 2$

Therefore,the required solution is $\frac{x^4}{4} - \frac{x^2}{2} = y + 2$

11. **(d)** $x^2 - 1 = C\left(1 + y^2\right)$

Explanation: We have,

xdx + ydy = x^2y dy - y^2x dx

x dx + y^2x dx = x^2y dy - y dy

$x\left(1 + y^2\right) dx = y\left(x^2 - 1\right) dy$

$\frac{xdx}{x^2-1} = \frac{ydy}{1+y^2}$

$\int \frac{xdx}{x^2-1} = \int \frac{ydy}{1+y^2}$

$\frac{1}{2}\int \frac{2xdx}{x^2-1} = \frac{1}{2}\int \frac{2ydy}{1+y^2}$

$\frac{1}{2}\log\left(x^2-1\right) = \frac{1}{2}\log\left(1+y^2\right) + \log c$

$\log\left(x^2-1\right) = \log\left(1+y^2\right) + \log c$

x^2 - 1 = $\left(1+y^2\right)c$

12. **(c)** $\frac{\pi}{2}$

Explanation: I = $\int\limits_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$

= $\int\limits_{\frac{-\pi}{2}}^{0} \frac{1}{1+e^{\sin x}} dx + \int\limits_{0}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$

= $\int\limits_{0}^{\frac{\pi}{2}} \left(\frac{1}{1+e^{\sin x}} + \frac{1}{1+e^{-\sin x}}\right) dx$

= $\int\limits_{0}^{\frac{\pi}{2}} \frac{1+e^{\sin x}}{1+e^{\sin x}} dx = \frac{\pi}{2}$

13. **(a)** $x^3 + y^3 = 12x + C$

Explanation: We have ,

$x^2 + y^2 \frac{dy}{dx} = 4$

$y^2 \frac{dy}{dx} = 4 - x^2$

$y^2 dy = \left(4 - x^2\right) dx$

$\Rightarrow \int y^2 dy = \int \left(4 - x^2\right) dx$

$\Rightarrow \frac{y^3}{3} = 4x - \frac{x^3}{3}$ + C

$\Rightarrow y^3 + 12x - x^3 + C$

$\Rightarrow x^3 + y^3 = 12x + C$

14. **(d)** 6

Explanation: $\lim\limits_{x \to 0} \frac{\sin 3x}{x} = \frac{k}{2}$

$\lim\limits_{x \to 0} \frac{\sin 3x}{3x} \times 3 = \frac{k}{2}$

$3 = \frac{k}{2}$

k = 6

15. **(c)** 14^4

Explanation: $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

|A| = 14 det(adjA)= $\det(A)^{3-1} = \det(A)^2$.Here the operation is done two times.so,

det (adj(adj A)) = $|A|^{(n-1)^2}$

det (adj(adj A)) = $14^{(3-1)^2} = 14^4$

16. **(a)** $\frac{1}{\sqrt{2}}$

Explanation: The given differential eqn. is

$\frac{dy}{\sqrt{1-y^2}} + \frac{dx}{\sqrt{1-x^2}} = 0 \Rightarrow$ $\sin^{-1} y + \sin^{-1} x = c$

At x = $\frac{1}{2}$, y = $\frac{\sqrt{3}}{2} \Rightarrow c = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} y = \cos^{-1} x$

Hence, y $\left(-\frac{1}{\sqrt{2}}\right)$ = sin $\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$

= $\sin\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) = \frac{1}{\sqrt{2}}$

17. **(a)** $\sqrt{\frac{2}{3}}$

Explanation: sin [$\cot^{-1}$ (cos $\frac{\pi}{4}$)] = sin [$\cot^{-1} \frac{1}{\sqrt{2}}$] = sin $\left[\sin^{-1} \sqrt{\frac{2}{3}}\right] = \sqrt{\frac{2}{3}}$.

Which is the required solution.

18. **(a)** perpendicular to z-axis

Explanation: We have,

$\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$

Also, the given line is parallel to the vector $\vec{b} = 3\hat{i} + \hat{j} + 0\hat{k}$

Let $x\hat{i} + y\hat{j} + z\hat{k}$ be perpendicular to the given line.

Now,

3x + 4y + 0z = 0

It is satisfied by the coordinates of z-axis, i.e. (0, 0, 1)

Hence, the given line is perpendicular to z-axis.

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true but R is not the correct explanation of A.

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

LHL = $\lim\limits_{x \to 2^-}$ f(x) = $\lim\limits_{h \to 0}$ f(2 - h)

= $\lim\limits_{h \to 0}$ [2 - h - 1] + |2 - h - 2|

= $\lim\limits_{h \to 0}$ [1 - h] + |-h| = $\lim\limits_{h \to 0}$ (0 + h) = 0

and f(2) = [2 - 1] + |2 - 2| = [1] + 0 = 1

$\therefore$ LHL $\neq$ f(2)

$\Rightarrow$ f(x) is discontinuous at x = 2.

Reason:

$Lf'(2) = \lim\limits_{h \to 0} \frac{f(2-h)-f(2)}{-h}$

$= \lim\limits_{h \to 0} \frac{[2-h-1]+|2-h-2|-[2-1]-|2-2|}{-h}$

$= \lim\limits_{h \to 0} \frac{0+h-1-0}{-h}$ [$\because \lim\limits_{h \to 0}$ [1 - h] = 0]

$= \lim\limits_{h \to 0} \left(1 - \frac{1}{h}\right) = -\infty$ (not defined)

$\therefore$ f(x) is not differentiable at x = 2.

Hence, both Assertion and Reason are true and Reason is not a correct explanation of Assertion.

Section B

21. We have, $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y + y\cos y}$

$\Rightarrow$ (sin y + y cos y) = x (2 log x + 1) dx

Integrating both sides, we get

$\int$ (sin y + y cos y) dy = $\int$ x (2 log x + 1) dx

$\Rightarrow \int \sin y dy + \int y \cos y dy = 2\int x \log x dx + \int x dx$

$\Rightarrow -\cos y + \left[y \int \cos y dy - \int \left\{\frac{d}{dy}(y) \int \cos y dy\right\} dy\right] = 2\left[\log x \int x dx - \int \left\{\frac{d}{dx}(\log x) \int x dx\right\} dx\right] + \frac{x^2}{2}$

$\Rightarrow -\cos y + [y \sin y - \int \sin y dy] = 2\left[\log x \times \frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2}\right] + \frac{x^2}{2}$

$\Rightarrow$ - cos y + y sin y + cos y = x^2 log x $-\frac{x^2}{2} + \frac{x^2}{2} + C$

$\Rightarrow$ y sin y = x^2 log x + C

Therefore ,y sin y = x^2 log x + C is the required solution of the given differential equation.

22. It is given that $f(x) = \begin{cases} \sin x - \cos x, \text{ if } x \neq 0 \\ -1, \text{ if } x = 0 \end{cases}$

We know that f is defined at all points of the real line.

Let k be a real number.

Case I: k $\neq$ 0,

Then f(k) = sin k - cos k

$\lim\limits_{\mathbf{x} \to \mathbf{k}} \mathrm{f(x)} = \lim\limits_{\mathrm{x} \to \mathrm{k}} (\sin \mathrm{x} - \cos \mathrm{x}) = \sin \mathrm{k} - \cos\mathrm{k}$

$\therefore \lim\limits_{x \to \mathbf{k}} (x) = f(k)$

Thus, f is continuous at all points 'x' such that x $\neq$ 0.

Case II: k = 0

Then f(k) = f(0) = -1

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-}$ (sin x - cos x) = sin 0 - cos 0 = 0 - 1 = -1

$\lim_{x\to0^+} f(x) = \lim_{x\to0^+}$ (sin x - cos x) = sin 0 - cos 0 = 0 - 1 = -1

$\therefore \lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = f(0)$

Therefore, f is continuous at x = 0.

Therefore, f has no point of discontinuity.

23. Here $\alpha = 90^0, \beta = 135^0$ and $\gamma = 45^0$

Since direction cosines of a line making angles α, β, γ with the x, y and z - axes respectively are $\cos\alpha, \cos\beta, \cos\gamma$

Therefore, the direction cosines of the required line are:

$\cos 90^0 = 0; \cos 135^0 = \frac{-1}{\sqrt{2}}, \cos 45^0 = \frac{1}{\sqrt{2}}$

$\left[\because \cos 135^0 = \cos(180^0 - 45^0) = -\cos 45^0 = \frac{-1}{\sqrt{2}}\right]$

OR

Here,it is given that Cartesion equation of line

$\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$

Now we have to find: equation of line in vector form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_2} = \lambda$

where $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ is a point on the line and $\overrightarrow{\mathrm{b}} = \mathrm{b}_1\hat{\mathrm{i}} + \mathrm{b}_2\hat{\jmath} + \mathrm{b}_3\hat{\mathrm{k}}$ is a vector parallel to the line.

Explanation:

From the Cartesian equation of the line, we can find $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$

Here, $\vec{a} = 3\hat{\imath} - \hat{2}_{\jmath} + 6\hat{k}$ and $\overrightarrow{\mathrm{b}} = \hat{2}1 - 5\hat{\jmath} + 4\hat{\mathrm{k}}$

Thus,vector form:

$\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k} + \lambda(2\hat{\imath} - 5\hat{\jmath} + 4\hat{k})$

24. Let E_1 : Two headed coin,

E_2 : Biased coin

and E_3 : Unbiased coin.

Let A : Coin shows head.

Then P (E_1) = P (E_2) = P (E_3) = $\frac{1}{3}$

As the two-headed coin has head on both sides so it will show head only.

Also P($\frac{A}{E_1}$) = P (correct answer given that he knows) = 1

And P($\frac{A}{E_2}$) = P (coin shows head given that the coin is biased) = 75% = $\frac{75}{100} = \frac{3}{4}$

And P($\frac{A}{E_3}$) = P (coin shows head given that the coin is unbiased) = $\frac{1}{2}$

Now the probability that the coin is two headed, being given that it shows head, is P($\frac{E_1}{A}$).

By using Bayes' theorem, we have:

$\mathrm{P}(\mathrm{E}_1|\mathrm{A}) = \frac{\mathrm{P}(\mathrm{E}_1)\cdot\mathrm{P}(\mathrm{A}|\mathrm{E}_1)}{\mathrm{P}(\mathrm{E}_1)\cdot\mathrm{P}(\mathrm{A}|\mathrm{E}_1)+\mathrm{P}(\mathrm{E}_2)\cdot\mathrm{P}(\mathrm{A}|\mathrm{E}_2)+\mathrm{P}(\mathrm{E}_3)\cdot\mathrm{P}(\mathrm{A}|\mathrm{E}_3)}$

$= \frac{\frac{1}{3}\cdot1}{\frac{1}{3}\cdot1+\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{1}{2}}$

$= \frac{\frac{1}{3}}{\frac{1}{3}\left(1+\frac{3}{4}+\frac{1}{2}\right)}$

$= \frac{\frac{1}{9}}{\frac{9}{4}} = \frac{4}{9}$

$\therefore$ P($\frac{E_1}{A}$) = $\frac{4}{9}$

25. We know that $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

$\therefore \quad \tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$

$= \tan^{-1}\left\{2\cos\left(2\times\frac{\pi}{6}\right)\right\}$

$= \tan^{-1}\left(2\cos\frac{\pi}{3}\right) = \tan^{-1}(2\times\frac{1}{2}) = \tan^{-1}1 = \frac{\pi}{4}$

Section C

26. According to the question,

Maximize $Z = x + 2y$

Subject to the constraints

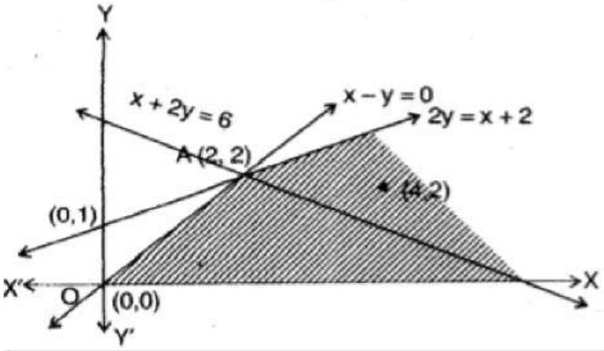
$x - y \geq 0$

$2y \leq x + 2$

such that $x \geq 0, y \geq 0$

We draw line $x - y = 0$ and $2y = x + 2$ and shaded the feasible region with respect to constraints sign.

We observe that the feasible region is unbounded and corner points are $O(0,0), A(2,2)$ only. Evaluating Z at all corner points,



Corner Points	Z = x + 2y
$(0,0)$	$Z = 0\ (Smallest)$
$(2,2)$	$Z = 6\ (largest)$

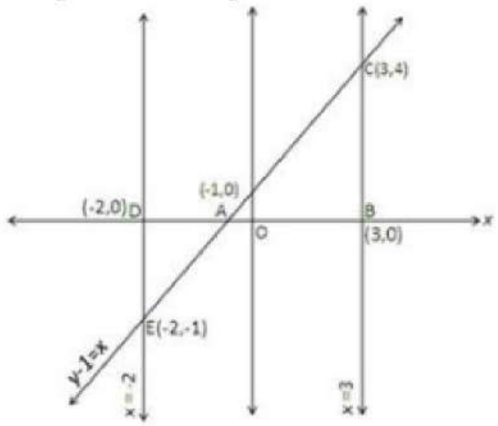
We are to determine the maximum value. As the feasible region is unbounded we can not say whether the largest value Z = 6 is maximum or not.

We draw the half plane $x + 2y > 6$ and notice that it has common points with feasible region. There does not exist any maximum value. For testing we let $x = 4, y = 2$, this point lie in the feasible region and at $(4,2)$ the value of $Z = 8$ i.e., greater than '6'.

27. p>To find area of region bounded by x-axis the ordinates x = -2 and x = 3 and y -1 = x ...(i)

Equation (i) is a line that meets at axes at (0, 1) and (-1, 0)

A rough sketch of the given information is as under:-



Bounded region provides the required area.

Now Required area = Area of Region ABCA + Area of Region ADEA

A = $\int_{-1}^{3} y dx + \left|\int_{-2}^{-1} y dx\right|$

$= \int_{-1}^{3}$ (x + 1)dx + |[-2^{-1} (x + 1)dx|

$= \left(\frac{x^2}{2} + x\right)_{-1}^{3} + \left|\frac{x^2}{2} + x\right|_{-2}^{-1}$

= [($\frac{9}{2}$ + 3) - ($\frac{1}{2}$ - 1)] + ($\frac{1}{2}$ - 1) - (2 - 2)]

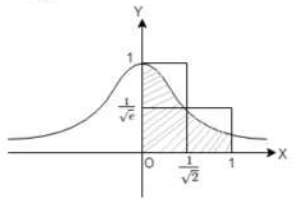
$= \left[\frac{15}{2} + \frac{1}{2}\right] + \left|-\frac{1}{2}\right|$

= 8 + $\frac{1}{2}$

A = $\frac{17}{2}$ sq. units

OR

As, $y=e^{-x^2}$



since, $x^2 \leq x$ when x$\in [0,1]$

$\Rightarrow\ -x^2 \geq -x$

$\Rightarrow\ e^{-x^2} \geq e^{-x}$

$\therefore\ \int_0^1 e^{-x^2}dx \geq \int_0^1 e^{-x}dx$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}S \geq -(e^{-x})_0^1 = 1-\frac{1}{e}\ .....(1)$

Also, $\int_0^1 e^{-x^2}dx \leq$ Area of the two rectangles

$\leq \left(1\times\frac{1}{\sqrt{2}}\right)+\left(1-\frac{1}{\sqrt{2}}\right)\times\frac{1}{\sqrt{e}}$

$\leq \left(1\times\frac{1}{\sqrt{2}}\right)+\left(1-\frac{1}{\sqrt{2}}\right)\times\frac{1}{\sqrt{e}}$

$\leq \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right) ....(2)$

From (1) and (2), we conclude that

$1-\frac{1}{e} \leq S \leq \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

If S be the area of the region enclosed by $y=e^{-x^2}$, y = 0 and x = 1, then $1-\frac{1}{e} \leq S \leq \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

28. We have to show $\int_0^a f(a-x)dx$ = $\int_0^a f(x)dx$

Take I = $\int_0^a f(a-x)dx$ ......(i)

Let a - x = z ⇒ -dx = dz ⇒ dx - dz

Also, x = 0 ⇒ z = a and x = a ⇒ z = 0

∴ I = $\int_a^0 f(z)(-dz)$ = $\int_a^0 f(z)dz$

⇒ I = $\int_0^a f(z)dz$ (∵ $\int_a^b f(x)dx$ = $-\int_0^a f(x)dx$

⇒ I = $\int_0^a f(x)dx$ .....(ii) (∵ $\int_a^b f(z)dz$ = $\int_a^b f(x)dx$

From (i) and (ii), we get

$\int_0^a(a-x)dx$ = $\int_0^a f(x)dx$

Now, $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}$ dx ......(iii)

⇒ I = $\int_0^{\pi/2}\frac{\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{x}{2}-x\right)+\cos\left(\frac{x}{2}-x\right)}dx$ = $\frac{\pi}{2\sqrt{2}}\int\frac{1}{\cos x\frac{1}{\sqrt{2}}+\sin x\frac{1}{\sqrt{2}}}dx$

= $\frac{\pi}{2\sqrt{2}}\int_0^{\frac{\pi}{2}}\frac{1}{\sin\ x\ \sin\frac{\pi}{4}+\cos\ x\ \cos\frac{\pi}{4}}dx$

= $\frac{\pi}{2\sqrt{2}}\int_0^{\frac{\pi}{2}}\frac{1}{\cos(x-\frac{\pi}{4})}dx$

2I = $\frac{\pi}{2\sqrt{2}}\int_0^{\frac{\pi}{2}}\sec(x+\frac{\pi}{4})dx$

= $\frac{\pi}{2\sqrt{2}}\ [\log(\sec\ x+\tan x)]_0^{\frac{\pi}{2}}$

= $\frac{\pi}{2\sqrt{2}}$ [log (sec $\frac{\pi}{2}$ + tan $\frac{\pi}{2}$) - [log (sec 0 + tan 0)]

2I = $\frac{\pi}{2\sqrt{2}}$ [∞ - log 1] = ∞

I = ∞

OR

Let the given integral be,

$I=\int_0^1\frac{1}{x^4+2x^3+2x^2+2x+1}dx$

$=\int_0^1\frac{1}{(x+1)^2(x^2+1)}dx$

$=\int_0^1\left\{-\frac{x}{2(x^2+1)}+\frac{1}{2(x+1)}+\frac{1}{2(x+1)^2}\right\}dx$

$=\int_0^1\frac{x}{2(x^2+1)}dx+\int_0^1\frac{1}{2(x+1)}dx+\int_0^1\frac{1}{2(x+1)^2}dx$

$= \left\{\frac{\log(x^2+1)}{4}\right\}_0^1 + \left\{\frac{\log(x+1)}{2}\right\}_0^1 - \left\{\frac{1}{2(x+1)}\right\}_0^1$

$= \frac{\log 2}{4} + \frac{\log 2}{2} - \frac{1}{4} + \frac{1}{2}$

$= \frac{\log 2}{4} + \frac{1}{4}$

$= (1/4)\log(2e)$

29. We have, $x = py + q \Rightarrow y = \frac{x-q}{p}$ ...(i)

And $z = ry + s \Rightarrow y = \frac{z-s}{r}$ ...(ii)

$\Rightarrow \frac{x-q}{p} = \frac{y}{1} = \frac{z-s}{r}$ [Using Eqs. (i) and (ii)] ...(iii)

Similarly, $\frac{x-q'}{p'} = \frac{y}{1} = \frac{z-s'}{r'}$ ...(iv)

From Eqs. (iii) and (iv),

$a_1 = p, b_1 = 1, c_1 = r$

and $a_2 = p', b_2 = 1, c_2 = r'$

if these given lines are perpendicular to each other, then

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow pp' + 1 + rr' = 0$

Which is the required condition.

OR

Suppose A, B and C be the three points with position vectors $2\hat{i} - \hat{j} + \hat{k}, -\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$ respectively.

We have to find the equation of a line passing through point A and parallel to $\overrightarrow{BC}$.

Clearly, $\overrightarrow{BC}$ = $(\hat{i} + 2\hat{j} + 2\hat{k}) - (-\hat{i} + 4\hat{j} + \hat{k})$ = $2\hat{i} - 2\hat{j} + \hat{k}$

We know that the equation of a line passing through a point $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Here, $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$.

Thus the vector equation of the required line is

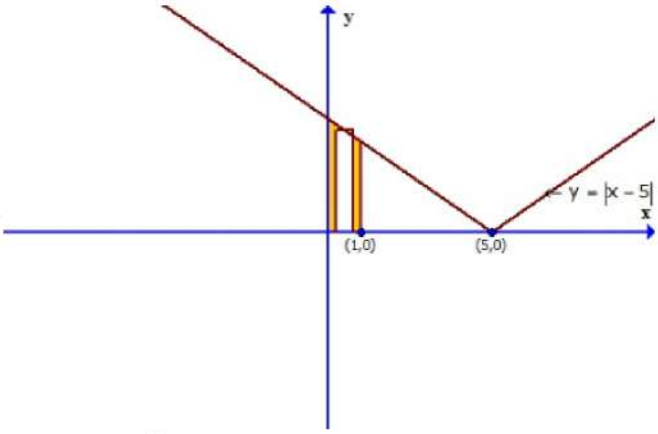
$\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ ...(i)

Reduction to cartesian form: substitute $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in (i), we get

$x\hat{i} + y\hat{j} + z\hat{k}$ = $(2 + 2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (1 + \lambda)\hat{k}$

$\Rightarrow$ x = 2 + 2λ, y = -1 - 2λ, z = 1 + λ

$\Rightarrow \frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$, which is the cartesian equivalent of equation (i).

30. 

We are given that

$y = |x-5|$ intersect x = 0 and x = 1 at (0, 5) and (1, 4)

Now, $y = |x-5|$

= - (x - 5) For all a ∈ (0, 1)

Integration represents the area enclosed by the graph from x = 0 to x = 1

Now area denoted by A , is given by

$A = \int_0^1 |y| dx$

$= \int_0^1 |x-5| dx$

$= \int_0^1 -(x-5) dx$

$= -\int_0^1 (x-5) dx$

$= -\left[\frac{x^2}{2} - 5x\right]_0^1$

$= -\left[\left(\frac{1}{2} - 5\right) - (0 - 0)\right]$

$= -\left(-\frac{9}{2}\right)$

$= \frac{9}{2}\text{sq}\cdot\text{ units}$

31. $y = a\{x + \sqrt{x^2+1}\}^n + b\{x - \sqrt{x^2+1}\}^{-n}$,

$\frac{dy}{dx} = \text{na}\{x + \sqrt{x^2+1}\}^{n-1}\left[1 + x\left(x^2+1\right)^{-\frac{1}{2}}\right] - nb\{x - \sqrt{x^2+1}\}^{-n-1}\left[1 - x\left(x^2+1\right)^{-\frac{1}{2}}\right]$

$\frac{dy}{dx} = \frac{\text{na}}{\sqrt{x^2+1}}\{x + \sqrt{x^2+1}\}^n + \frac{nb}{\sqrt{x^2+1}}\{x - \sqrt{x^2+1}\}^{-n}$

$\frac{dy}{dx} = \frac{n}{\sqrt{x^2+1}}\left[a\{x + \sqrt{x^2+1}\}^n + b\{x - \sqrt{x^2+1}\}^{-n}\right]$

$x\frac{dy}{dx} = \frac{nx}{\sqrt{x^2+1}}y$

$\frac{d^2y}{dx^2} = \frac{nx}{\sqrt{x^2+1}}\frac{dy}{dx} + y\left[\frac{\sqrt{x^2+1} - x^2\left(x^2+1\right)^{-\frac{1}{2}}}{x^2+1}\right]$

$\frac{d^2y}{dx^2} = \frac{n^2x^2}{x^2+1} + y\left[\frac{1}{\left(x^2+1\right)\sqrt{x^2+1}}\right]$

$\frac{d^2y}{dx^2} = \frac{n^2x^2(\sqrt{x^2+1})+y}{\left(x^2+1\right)\sqrt{x^2+1}}$

$\left(x^2-1\right)\frac{d^2y}{dx^2} = \frac{n^2x^4(\sqrt{x^2+1})+x^2y}{\left(x^2+1\right)\sqrt{x^2+1}} - \frac{n^2x^2(\sqrt{x^2+1})+y}{\left(x^2+1\right)\sqrt{x^2+1}}$

Now,

$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - ny$

$= \frac{n^2x^4(\sqrt{x^2+1})+x^2y}{\left(x^2+1\right)\sqrt{x^2+1}} - \frac{n^2x^2(\sqrt{x^2+1})+y}{\left(x^2+1\right)\sqrt{x^2+1}} + \frac{nx}{\sqrt{x^2+1}}y - ny$

= 0

LHS = RHS.

Hence Proved.

Section D

32. According to the question, $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ ...(i)

and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ ...(ii)

Subtracting Eq. (ii) from Eq. (i),

$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = (\vec{c} \times \vec{d}) - (\vec{b} \times \vec{d})$

$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) + (\vec{b} \times \vec{d}) - (\vec{c} \times \vec{d}) = \vec{0}$

$\Rightarrow \quad \vec{a} \times (\vec{b} - \vec{c}) + (\vec{b} - \vec{c}) \times \vec{d} = \vec{0}$

$\Rightarrow \quad \vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = \vec{0}$

$[\because \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}]$

$\therefore \quad (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0}$

$[\because \vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$, given]

The cross-product of vectors $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ is a zero vector, so $\vec{a} - \vec{d}$ is parallel $\vec{b} - \vec{c}$.

33. Given, A = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Clearly, the co-factors of elements of |A| are given by,

$A_{11} = \cos\alpha$; $A_{12} = -\sin\alpha$; $A_{13} = 0$;

$A_{21} = \sin\alpha$; $A_{22} = \cos\alpha$; $A_{23} = 0$

$A_{31} = 0$; $A_{32} = 0$ and $A_{33} = 1$

$\therefore$ adj (A) = $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, A (adj A)

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{...(i)}$$

$$(\text{ adj } A) \cdot (A) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{...(ii)}$$

and $|A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= 1\cdot(\cos^2\alpha + \sin^2\alpha) = 1$ [expanding along R_3]

$$\therefore \quad |A| I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From qs. (i), (ii) and (iii), we get,

A (adj A) = (adj A).A = |A| I_3

OR

For the given system of equations, we have

D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix}$ = 1 × (10 - 9) - 1 × (5 - 3) +1 × (3 - 2) = 0,

D_1 = $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 7 & 3 & 5 \end{vmatrix}$ = 1 × (10 - 9) - 1 × (20 - 21) + 1 × (12 - 14) = 0,

D_2 = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 7 & 5 \end{vmatrix}$ = 1 × (20 - 21) - 1 × (5 - 3) + 1 × (7 - 4) = 0,

and, D_3 = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \end{vmatrix}$ = 1 × (14 -1 2) - 1 × (7 - 4) + 1 × (3 - 2) = 0.

Thus, we have D = D_1 = D_2 = D_3 = 0.

So, either the system is consistent with infinitely many solutions or it is inconsistent. Consider the first two equations, these equations can be written as

x + y = 1 - z

x + 2y = 4 - 3z

In order to solve these equations let us use Cramer's rule.

Here, D = $\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$ = 2 - 1 = 1, D_1 = $\begin{vmatrix} 1-z & 1 \\ 4-3z & 2 \end{vmatrix}$ = 2 - 2z - 4 + 3z = z - 2

and, D_2 = $\begin{vmatrix} 1 & 1-z \\ 1 & 4-3z \end{vmatrix}$ = 4 - 3z -1 + z = 3 - 2z.

$\therefore$ x = $\frac{D_1}{D}$ and y = $\frac{D_2}{D}$

$\Rightarrow$ x = z - 2, y = 3 - 2z.

Let z = k, where k is any real number. Then, we get

x = k - 2, y = 3 - 2k and z = k

Substituting these values in the LHS of third equation x + 3y + 5z = 7, we get

k - 2 + 3(3 - 2k) + 5k = k - 2 + 9 - 6k + 5k = 7 = RHS

Hence, these values satisfy the third equation.

Hence, x = k - 2, y = 3 - 2k , z = k is a solution of the given system of equation for every value of k.

34. Given that A = {2, 3, 4}, B = {2, 5, 6, 7}

1. Let $f: A \to B$ defined by
   f = {(x, y) : y = x + 3}
   i.e. f = {(2, 5), (3, -6), (4, 7)} f = {(2, 5), (3, 6), (4, 7)} which is an injective mapping.
2. Let $g: A \to B$ denote a mapping such that g = {(2, 2), (3, 5), (4, 5) which is not an injective mapping.
3. Let $h: B \to A$ denote a mapping such that h = {(2, 2), (5, 3), (6, 4), (7, 4)} which is a mapping from B to A.

OR

Given that, A = R - {3}, B = R - {1}.

$f: A \to B$ is defined by $f(x) = \frac{x-2}{x-3} \; \forall x \in A$

For injectivity

Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

So, f(x) is an injective function

For surjectivity

Let $y = \frac{x-2}{x-3} \Rightarrow$ x - 2 = xy - 3y

$\Rightarrow x(1 - y) = 2 - 3y \Rightarrow x = \frac{2-3y}{1-y}$

$\Rightarrow x = \frac{3y-2}{y-1} \in A, \; \forall y \in B$ [codomain]

So, f(x) is surjective function.

Hence, f(x) is a bijective function.

35. According to given question ,$I = \int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$ ...(i)

we know that ,

$\int_0^a f(x) = \int_0^a f(a-x) dx$

$\Rightarrow I = \int_0^\pi \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \tan(\pi - x)} dx$

$\Rightarrow I = \int_0^\pi \frac{-(\pi - x)\tan x}{-\sec x - \tan x} dx \;\; [\because tan(\pi - x) = -tanx, sec(\pi - x) = -secx]$

$\Rightarrow I = \int_0^\pi \frac{(\pi - x)\tan x}{\sec x + \tan x} dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2I = \int_0^\pi \frac{\pi \tan x}{\sec x + \tan x} dx$

$= \pi \int_0^\pi \frac{\tan x}{\sec x + \tan x} dx$

$= \pi \int_0^\pi \frac{\frac{sinx}{cosx}}{\frac{1}{cosx} + \frac{sinx}{cosx}} dx$

$= \pi \int_0^\pi \frac{\sin x}{1 + \sin x} dx$

$= \pi \int_0^\pi \frac{\sin x}{1 + \sin x} dx$

$= \pi \int_0^\pi \frac{1 + \sin x - 1}{1 + \sin x} dx$

$= \pi \int_0^\pi \left(\frac{1 + \sin x}{1 + \sin x} - \frac{1}{1 + \sin x} \right) dx$

$= \pi \int_0^\pi \left(1 - \frac{1}{1 + \sin x} \right) dx$

$= \pi \left[\int_0^\pi 1 dx - \int_0^\pi \frac{1}{1 + \sin x} dx \right]$

$= \pi \left[\int_0^\pi 1 dx - 2\int_0^{\pi/2} \frac{1}{1 + \sin x} dx \right] \left[\because \int_0^{2a} f(x) = 2\int_0^a f(x) dx \;\; \text{if} \;\; f(2a - x) = f(x) \;\; \right]$

$= \pi \left[\int_0^\pi dx - 2\int_0^{\pi/2} \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx \right]$

$= \pi \left[[x]_0^\pi - 2\int_0^{\pi/2} \frac{1 - \sin x}{\cos^2 x} dx \right] \; [\because \sin^2 x + \cos^2 x = 1 \Rightarrow 1 - \sin^2 x = \cos^2 x]$

$= \pi \left[[x]_0^\pi - 2\int_0^{\pi/2} \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right) dx \right]$

$= \pi \left[(\pi - 0) - 2\int_0^{\pi/2} \left(\sec^2 x - \sec x \tan x \right) dx \right]$

$= \pi \left\{ \pi - 2[\tan x - \sec x]_0^{\pi/2} \right\}$

$= \pi \left\{ \pi - \lim_{x \to \frac{\pi}{2}} 2(\tan x - \sec x) + 2(\tan 0 - \sec 0) \right\}$

$2I = \pi\{\pi - 0 + 2(0-1)\} = \pi(\pi - 2)$

$\therefore \quad I = \frac{\pi}{2}(\pi - 2)$

Section E

36. **Read the text carefully and answer the questions:**

The Government declare that farmers can get ₹300 per quintal for their onions on 1st July and after that, the price will be dropped by ₹3 per quintal per extra day. Govind's father has 80 quintals of onions in the field on 1st July and he estimates that the crop is increasing at the rate of 1 quintal per day.

(i) Let x be the number of extra days after 1st July.

$\therefore$ Price = ₹(300 - 3 $\times$ x) = ₹(300 - 3x)

Quantity = 80 quintals + x(1 quintal per day) = (80 + x) quintals

Revenue = R(x) = Quantity $\times$ Price = (80 + x) (300 - 3x) = 24000 - 240x + 300x - $3x^2$

R(x) = 24000 + 60x - $3x^2$

(ii) We have, R(x) - 24000 + 60x - $3x^2$

$\Rightarrow$ R'(x) = 60 - 6x $\Rightarrow$ R''(x) = -6

For R(x) to be maximum, R'(x) = 0 and R''(x) < 0

$\Rightarrow$ 60 - 6x = 0 $\Rightarrow$ x = 10

(iii) Govind's father will attain maximum revenue after 10 days.

So, he should harvest the onions after 10 days of 1st July i.e., on 11th July.

OR

Maximum revenue is collected by Govind's father when x = 10

$\therefore$ Maximum revenue = R(10)

= 24000 + 60(10) - $3(10)^2$ = 24000 + 600 - 300 = ₹24,300

37. **Read the text carefully and answer the questions:**

On her birthday, Seema decided to donate some money to the children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹).

(i) 5x - 4y = 40

5x - 8y = -80

$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

(ii) A = $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B = $\begin{bmatrix} 40 \\ -80 \end{bmatrix}$

|A| = -40 + 20 = -20 $\neq$ 0

Cofactor matrix A = $\begin{bmatrix} -8 & -5 \\ 4 & 5 \end{bmatrix}$ adj A = $\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$

(iii) A = $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B = $\begin{bmatrix} 40 \\ -80 \end{bmatrix}$

|A| = -40 + 20 = -20 $\neq$ 0

Cofactor matrix A = $\begin{bmatrix} -8 & -5 \\ 4 & 5 \end{bmatrix}$, adj A = $\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$

X = A^{-1} B ...(i)

$A^{-1} = \frac{1}{|A|} \cdot adjA$

$A^{-1} = \frac{1}{-20} \cdot \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$

From (i)

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-20} \cdot \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-20} \begin{bmatrix} -320 - 320 \\ -200 - 400 \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$

x = 32 and y = 30

OR

There are 32 Children, and each child is given ₹30.

Total money spent by Seema = 32 $\times$ 30 = ₹960

Hence Seema spends ₹960 in distributing the money to all the students of the Orphanage.

38. **Read the text carefully and answer the questions:**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in all subjects) = P(M $\cap P \cap$ C)

= P(M) $\times$ P(P) $\times$ P(C)

= 0.2 $\times$ 0.3 $\times$ 0.5 = 0.03

(ii) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in on subjects) = $P(\overline{M} \cap \overline{P} \cap \overline{C})$

= $P(\overline{M}) \times P(\overline{P}) \times P(\overline{C})$

= 0.8 $\times$ 0.7 $\times$ 0.5 = 0.280